You do not have to simplify these numerical answers, but you must present your answer using only numbers combined by the operations of addition, subtraction, multiplication, division, powers and factorials.

1. The 14-character string "NONRESTRICTIVE" contains two each of the following five characters: ‘E’, ‘I’, ‘N’, ‘R’ and ‘T’. The remaining characters occur one time each.

(a) How many different strings can be made by arranging the characters of this string?

\[
\text{Ans: } \frac{14!}{2!2!2!2!}
\]

(b) Out of all the possible strings from part (a), how many contain none of the following as substrings: "EE", "II", "NN", "RR" and "TT"?

\[
\text{Ans: } N \text{ is the number in part (a). Using the usual conditions, } S_1 = 5N(c_1) = 5 \cdot \frac{13!}{2!2!2!2!}, \quad S_2 = 10 \cdot \frac{12!}{2!2!2!}, \quad S_3 = 10 \cdot \frac{11!}{2!2!}, \quad S_4 = 5 \cdot \frac{10!}{2!} \text{ and } S_5 = 9!. \quad \text{So}
\]
\[
N(c_1c_2c_3c_4c_5) = \frac{14!}{32} - 5 \cdot \frac{13!}{16} + 10 \cdot \frac{12!}{8} - 10 \cdot \frac{11!}{4} + 5 \cdot \frac{10!}{2} - 9!.
\]

(c) Of all the possible strings from part (a), how many contain at least one of the five strings in part (b) as a substring?

\[
\text{Ans: } L_1 = S_1 - S_2 + S_3 - S_4 + S_5 = 5 \cdot \frac{13!}{16} - 10 \cdot \frac{12!}{8} + 10 \cdot \frac{11!}{4} - 5 \cdot \frac{10!}{2} + 9!.
\]

(d) Of all the possible strings from part (a), how many contain exactly one of the five strings in part (b) as a substring?

\[
\text{Ans: } E_1 = S_1 - 2S_2 + 3S_3 - 4S_4 + 5S_5 = 5 \cdot \frac{13!}{16} - 20 \cdot \frac{12!}{8} + 30 \cdot \frac{11!}{4} - 20 \cdot \frac{10!}{2} + 5 \cdot 9!.
\]

2. (a) How many arrangements of the 8-letter string "MANIFEST" have no letters in their original positions?

\[
\text{Ans: } d_8 = 8! \left( \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} \right).
\]

(b) How many arrangements of that string has exactly three of the letters in their original positions?

\[
\text{Ans: } \binom{8}{3} d_5 = \frac{8!}{3!5!} \left( \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right).
\]