1. (20%) Find the value of $\det A$ for the matrix $A$ shown.

$$
A = \begin{bmatrix}
2 & 2 & 0 & 0 & 2 \\
1 & 1 & -6 & 0 & -5 \\
2 & 0 & -4 & 3 & 2 \\
2 & 0 & -1 & 3 & -3 \\
0 & 4 & 0 & 0 & -4
\end{bmatrix}
$$

2. (20%) Using orthogonal matrix and diagonalization, identify and graph (to scale) the conic section

$$
9x^2 + 24xy + 16y^2 - 4x + 3y = 10
$$

3. (30%) In an experiment performed on a specimen, the following correspondence was found between the applied force $F$ (in N) and the elongation $\delta$ (in mm):

<table>
<thead>
<tr>
<th>$F$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.5</td>
<td>3.3</td>
<td>4.5</td>
<td>5.5</td>
<td></td>
</tr>
</tbody>
</table>

Using matrix algebra, find the least square line (line of best fit)

$$
\delta = aF + b
$$

Use this line to estimate the value of $\delta$ for $F = 2.5$ N.

4. (30%) It is known that the eigenvalues for the matrix $A$ shown are $\lambda_1 > \lambda_2 > \lambda_3 = 1$. For this matrix $A$, determine $(a)$ the values of $\lambda_1$ and $\lambda_2$, $(b)$ the eigenvectors $K_1, K_2, K_3$ (using simplest integers for its entries), $(c)$ the modal matrix $M$, $(d)$ a square root $\sqrt{A}$.

$$
A = \begin{bmatrix}
-11 & 6 & 0 \\
30 & 16 & 20 \\
60 & -18 & 9
\end{bmatrix}
$$