The crank $AB$ rotates with a constant angular velocity $\omega_{AB} = 2 \text{ rad/s}$ and the slider $D$ moves in a circular groove. Determine the angular velocity $\omega_{BD}$ of link $BD$ and the velocity $v_D$ of slider $D$ when it moves downward and crosses the line $AO$.

The constraint condition is $\overline{AB} + \overline{BD} + \overline{DO} = \overline{AO}$. We write

\[1(-\cos \theta_1 \mathbf{i} + \sin \theta_1 \mathbf{j}) + 2(\cos \theta_2 \mathbf{i} + \sin \theta_2 \mathbf{j}) + 3(\cos \theta_3 \mathbf{i} - \sin \theta_3 \mathbf{j}) = 4 \mathbf{i} \quad (1)\]

\[-\cos \theta_1 + 2 \cos \theta_2 + 3 \cos \theta_3 = 4 \quad (1')\]

\[\sin \theta_1 + 2 \sin \theta_2 - 3 \sin \theta_3 = 0 \quad (2)\]

\[-(1 - \frac{1}{2} \theta_1^2) + 2 (1 - \frac{1}{2} \theta_2^2) + 3 (1 - \frac{1}{2} \theta_3^2) = 4 \quad (1'')\]

\[\theta_1 + 2 \theta_2 - 3 \theta_3 = 0 \quad (2'')\]

\[\theta_1^2 - 2 \theta_2^2 - 3 \theta_3^2 = 0 \quad (1''')\]

\[\theta_3 = \frac{1}{3} (\theta_1 + 2 \theta_2) \quad (2'''')\]

\[\dot{\theta}_2 = 0.2 \left( -1 \pm \sqrt{6} \right) \dot{\theta}_1 \quad \dot{\theta}_2 = 0.2 \left( -1 \pm \sqrt{6} \right) \dot{\theta}_1 \quad \dot{\theta}_3 = \frac{1}{3} (\dot{\theta}_1 + 2 \dot{\theta}_2) \quad (2''''')\]

Since $\omega_{AB} = 2 \text{ rad/s}$ and $\dot{\theta}_2$ must be negative, we write $\dot{\theta}_1 = -2 \text{ rad/s}$ and $\dot{\theta}_2 = -0.5798 \text{ rad/s}$, $\dot{\theta}_3 = -1.0532 \text{ rad/s}$, \(v_D = \overline{OD} \dot{\theta}_3 = -3.1596 \text{ m/s}\). Thus,

\[\omega_{BD} = 0.580 \text{ rad/s} \quad (2)\]

\[v_D = 3.16 \text{ m/s} \quad (2)\]