A spacecraft $S$ approaches Venus along a hyperbolic trajectory $SQ$ of eccentricity $\varepsilon = 2$ as shown. As it reaches $Q$, retrorockets are fired momentarily to insert it into an elliptic orbit as indicated. If the mass of Venus is $0.8144$ times the mass of the Earth, determine for the spacecraft (a) its speed as it approaches $Q$, (b) its speed after firing of retrorockets at $Q$, (c) the time required to travel from $Q$ to $P$.

Let kilometer (km) and seconds (s) be used in the calculations.

\begin{align*}
(a) \quad & \frac{1}{r_Q} = \frac{GM_V}{h_1^2} (1 + \varepsilon \cos \theta) = \frac{GM_E (M_V/M_E)}{r_Q^2 (v_Q)^2_{hyp}} (1 + \varepsilon \cos \theta), \quad GM_E = g_E R_E^2 \\
& \frac{1}{16 \times 10^3} = \frac{(9.81 \times 10^{-3}) (6370)^2 (0.8144)}{(16 \times 10^3)^2 (v_Q)^2_{hyp}} (1 + 2 \cos 0) \quad (v_Q)_{hyp} = 7.80 \text{ km/s} \tag{4}

(b) \quad & \frac{1}{r_P} + \frac{1}{r_Q} = \frac{2GM_V}{h_2^2} = \frac{2GM_E (M_V/M_E)}{r_Q^2 (v_Q)^2_{ell}} \\
& \frac{1}{10 \times 10^3} + \frac{1}{16 \times 10^3} = \frac{2(9.81 \times 10^{-3}) (6370)^2 (0.8144)}{(16 \times 10^3)^2 (v_Q)^2_{ell}} \quad (v_Q)_{ell} = 3.94785 \text{ km/s} \quad (v_Q)_{ell} = 3.95 \text{ km/s} \tag{3}

(c) \quad & t_{QP} = \frac{\tau}{2} = \frac{\pi (r_P + r_Q) \sqrt{r_P r_Q}}{2h_2} = \frac{\pi [(10 + 16) \times 10^3] \sqrt{10} (16) \times 10^3}{2(16 \times 10^3)(3.94785)} \quad s = 8178.5 \text{ s} \quad t_{QP} = 2.27 \text{ hours} \tag{3}
\end{align*}