• Q: Is identity (sameness) a relation? A relation between objects or names? Frege earlier argued that identity is a relation between names. Why?

◦ ‘a=a’ is analytic and knowable a priori, but ‘a=b’ is synthetic and knowable only a posteriori. The two statements differ in their cognitive value. We can understand the first to be true without understanding the second to be true. We can know the first one to be true without any investigation, but not so for the second.

What one wishes to express with “a=b” seems to be that the signs or names ‘a’ and ‘b’ name the same thing; and in that case we would be dealing with those signs: a relation between them would be asserted.

◦ Here is an argument that identity holds for names, not objects: Synthetic a posteriori statements, like ‘a=b’, are always contingent. So, it is possible that a does not equal b. But if identity holds of objects, ‘a=a’ and ‘a=b’ would each be necessarily true. So, ‘a=b’ does not tell us that some object is identical to itself, but that ‘a’ and ‘b’ happen to refer to the same thing. And this is a contingent truth — they could have referred to different things.

• Terms:

analytic: That which is true (or false) in virtue of the meanings of words alone.
synthetic: That which is not true (or false) in virtue of the meanings of words alone.

a priori: That which is knowable prior to, or independent of, experience.
a posteriori: That which is knowable only through experience.

necessary: That which is true (or false) in all possible worlds.
contingent: That which is true in some possible worlds, but false in others.
Frege no longer accepts that identity is a relationship between names, as argued above. This is because:

This connection [between sign and object], however, is arbitrary . . . Hence, a sentence like “a=b” would no longer refer to a matter of fact but rather to our matter of designation; no genuine knowledge would be expressed by it.

We can have two (or more) modes of presentation for the same referent. Frege uses this idea of sense as his solution to the puzzle over identity statements.

Example: We can designate the same point in a triangle under the following two modes of presentation: ‘point of intersection of a and b’ and ‘point of intersection of b and c.’

Similarly, ‘a’ and ‘b’ might offer two modes of presentation for the same referent. We can distinguish the sense of a sign (e.g., a proper name), which is the mode of presentation, from the reference of the sign (i.e., the nominatum). ‘Evening Star’ and ‘morning star’ differ in sense, but they have the same reference. Other words have a sense, but no reference — e.g., ‘the series with the least convergence.’ Synonyms, even across languages, have the same sense. Though, a word’s sense might be context-dependent.

Those who understand a word or sentence grasp its sense. But note that one object can fall under many senses, and complete knowledge of the object would require knowledge of all these senses:

The sense of a proper name is grasped by everyone who knows the language or the totality of designations of which the proper name is a part; this, however illuminates the nominatum, if there is any, in a very one-sided fashion. A complete knowledge of the nominatum would require that we could tell immediately in the case of any given sense whether it belongs to the nominatum. This we shall never be able to do. [Also, see footnote 2.]

Sense determines reference, according to Frege.

Frege notes the use/mention distinction we have already discussed when he writes:

When words are used in the customary manner then what is talked about are their nominata. But it may happen that one wishes to speak about the words themselves or about their senses.
Example: Quotation.

Frege’s concept of indirect discourse:

In indirect (oblique) discourse we speak of the sense, e.g., of the words of someone else. From this it becomes clear that also in indirect discourse words do not have their customary nominata; they here name what customarily would be their sense.

Distinguish:

indirect speech: talk regarding the sense of words  
customary speech: talk regarding the reference of words

• The sense and nominatum of a word (sign) are to be distinguished from the ideas (image) the word generates in our minds. This is a denial of Locke’s theory of meaning. Frege’s senses are objective. Frege’s senses are part of our “common treasure of thoughts” and are not to be found in individual minds.

The image thereby differs essentially from the connotation of a sign, which latter may well be common property of many and is therefore not a part or mode of the single person’s mind; for it cannot well be denied that mankind possesses a common treasure of thoughts which is transmitted from generation to generation.

• Frege claims, metaphorically, that senses are “in between” our ideas and the object designated.

The nominatum of a proper name is the object itself which is designated thereby; the image which we may have along with it is quite subjective; the sense lies in between, not subjective as in the image, but not the object either.

He provides a simile: it is like looking at the moon through a telescope. There is the moon (nominatum), the “real image” on the telescope’s lens (sense), and the retinal image (idea).

Words, expressions, and sentences can differ in 3 corresponding ways: the object they designate, their sense, or the ideas they generate in our minds.

The difference of a translation from the original should properly not go beyond the first level [i.e., a difference in imagery/ideas generated]. Among the differences possible in this connection we mention the shadings and colorings which poetry seeks to impart to the senses. These shadings and colorings are not objective.
Every listener or reader has to add them in accordance with the hints of the poet or speaker. Surely, art would be impossible without some kinship among human imageries; but just how far the intentions of the poet are realized can never be exactly ascertained.

- Terminology: a sign expresses its sense and designates its nominatum.

  ○ A sign may lack a nominatum, and one might be skeptical about the existence of mind-independent nominata in general, but we generally presume that there is some object out there in the world corresponding to terms like ‘moon’.

- Just as words have sense and nominata, whole declarative sentences have sense and nominata as well. The sense of a sentence is the thought or proposition it expresses. [Note: Such thoughts are not psychological entities that are in our heads — like you might speak of conscious thoughts. Instead, these thoughts are shared by all who understand the sentences.] The morning star/evening star propositions are supposed to illustrate this.

  ○ Frege distinguishes contexts in which we are concerned with “questions of truth” from contexts in which we are concerned only with “artistic appreciation”. The former contexts prompt us to search for nominata for whole sentences.

  ○ The nominatum of a sentence is its truth-value (i.e., the True or the False). These truth-values are objects — just like the Sun! All true sentences have the same nominatum, and all false sentences have the same nominatum.

  ○ Truth-value is preserved when co-referential terms (with different senses, say) are substituted.

- The nominatum of a sentence is its truth-value, and this remains unaltered when we substitute in co-referential terms.

  ○ Leibniz’s salva veritate principle, Compositionality of Sense, and Compositionality of Reference.

  ○ Does this hold when we substitute in whole sentences?

    A sentence in direct discourse nominates again a sentence but in indirect discourse it nominates a proposition.

  ○ It fails to hold for the case of ‘that’ clauses — in these cases, the nominatum is a proposition, not a truth-value.
In this case then the clause has as its nominatum a proposition, not a truth-value; its sense is not a proposition but it is the sense of the words “the proposition that...”, which is only a part of the proposition corresponding to the total sentence-structure. This occurs in connection with ‘to say’, ‘to hear’, ‘to opine’, ‘to be convinced’, ‘to infer’, and similar words.

○ In these cases, the truth of the whole statement does not depend on the truth of the ‘that’ clause. This is a reason to think that the nominatum of the clause is not a truth-value.

The only correct conclusion is that the nominatum of a sentence is not always its truth-value, and that ‘morning star’ does not always nominate the planet Venus; for this is indeed not the case when the word is used with its indirect nominatum.

● Our actions depend on our beliefs — not the truth of what is believed:

When Wellington, toward the end of the battle of Belle-Alliance was glad that the Prussians were coming, the ground of his rejoicing was a conviction. Had he actually been deceived, he would not have been less glad, as long as his belief persisted; and before he arrived at the conclusion that the Prussians were coming he could not have been glad about it, even if in fact they were already approaching.

○ The same point holds for our inferences.

● Note Frege’s discussion of the “Kepler died in misery” example. About such examples, Frege claims that there is a presupposition that Kepler meets the description ‘he who discovered the elliptical shape of the planetary orbits’, but the presupposition is not a part of the sentence.

○ Just as there are words that lack a referent, there are sentences that lack a referent. That is, there are sentences that are neither true nor false. Frege alleges that sentences containing a word or expression lacking a referent are themselves neither true nor false. In this, we see that Frege disagrees with Russell. Whereas Russell would say “The present King of France is bald” is false, Frege would say it has no reference — it is neither true nor false.

● Note Frege’s discussion of “illumination”:

... but the illumination might then easily appear inappropriate, just as if one were to sing a song of sad content in a cheerful manner.