Double Auction Performance in a Circular Flow Economy

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Abstract: Double auction markets have consistently been shown to realize almost full efficiency and prices very near the theoretical prediction. Research typically focuses on the performance of the double auction in single market environments. However, real economic activity is coordinated across multiple markets, a far more complex task. This paper uses experimental methods to examine the double auction’s performance in a circular flow economy where two markets are operating simultaneously. The results show that the double auction succeeds in achieving a high level of efficiency even though prices remain considerably more volatile than typical single market settings.

JEL Classification: C92; D44; D51; E00

Keywords: General equilibrium; Double auction; Circular flow economy

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The double auction (DA) institution is widely used in markets where information is decentralized. In the DA institution, a party interested in trading has information on the highest price at which a buyer stands ready to make a purchase and the lowest price at which a seller is offering to sell. Additionally, this institution typically provides participants with information about trading volume and transaction price history. An agent desiring to trade can either announce a proposed transaction price or can accept a price currently quoted from a party on the other side of the market. The ease with which a DA can be implemented has led to its employment in numerous markets. For example, both the NYSE and the CBOT use a form of the DA. The prevalence of this trading institution in the real world has been justified with its laboratory success of the DA. Starting with Smith (1962), researchers have continually found that the DA converges towards full efficiency and achieves a virtually uniform price after only a few replications under a wide variety of conditions.\footnote{It should be noted that Van Boening and Wilcox (1996) demonstrate that the DA’s success is not “independent of structure and strategy” by showing that efficiency is relatively low (~80%) when sellers have avoidable costs. Other exceptions have been found to exist when there are externalities [e.g. Plott (1983)] and when participants have market power [e.g. Holt, Langan, and Villamil (1986)].} In fact the DA has been found to outperform many other commonly used institutions. Ketcham et al. (1984) and Smith et al. (1982) compare posted offer and sealed bid offer markets with the DA respectively. Also, numerous researchers have used the DA to study more complex trading situations. Lim et al. (1994) use the DA in studying an overlapping generations model while Camerer and Kunreuther (1989) examine a market for insurance.

Most work with the double auction has remained in the realm of only one good. In reality markets do not operate in isolation, but rather agents operate in multiple markets simultaneously. Beyond completing a specific transaction, another function of
markets is to serve as a means for coordinating economic activity. Multiple market experiments typically select trading institutions based upon single market performance records. Deck et al. (2001) use a DA in experiments designed to examine the acceptance of fiat money. Noussair et al. (1995) study international trade and comparative advantage in a laboratory economy where trades are conducted through a multi-unit version of the double auction. However, an institution’s success in a simple market environment does not guarantee that the institution will succeed in more complex trading environments, such as those involving multiple commodities. Therefore, it is important to understand how an institution performs in these broader trading contexts. Some insight into multiple market performance has been provided by Williams, et al. (2000) who study a setting in which two markets operate simultaneously so that individuals who have an induced valuation over a two good bundle can trade with sellers who are endowed with units of each of the two goods. The results of these experiments indicate that buyers are able to achieve approximately 87% efficiency\(^2\) and that prices tend to be biased potentially due to a liquidity constraint. However, prices still tend to be tightly concentrated in those markets.

This paper reports the results of laboratory experiments conducted in a two good circular flow economy and evaluates the performance of the DA in such a setting. The results indicate that the DA is successful in achieving a high percentage of the gains from trade. However, prices remain more volatile in this environment than in the simple single market setting. The next section describes the double auction as it is implemented in this study. A separate section details the experimental design, which is followed by the results. A final section contains concluding remarks.

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\(^2\) In the design of Williams, et al. efficiency is only defined in the sense of pareto optimality.
Double Auction Institution

Small format variations to the DA can greatly influence market outcomes.\(^3\) Therefore, one must temper results with the design of the implemented mechanism. The computerized DA institution used in this study employed an improvement rule but did not maintain a bid or offer queue. While a bid/offer queue is common in many experimental studies, eliminating such a restraint allows agents to more readily adjust their behavior as additional information becomes available in this complicated two goods environment. Also, this structure coincides with rules 71 and 72 of the New York Stock Exchange as reported by Leffler and Farwell (1963).

In this version of the DA a seller (buyer) can announce to every market participant an ask (bid) price at which they were willing to sell (buy). In the experiments prices were quoted in US cents. At any time all traders can observe the maximum bid (minimum ask) price at which a unit could be sold (bought). To complete a transaction at this price a buyer (seller) simply accepts this ask (bid). A further refinement on trading is the use of an improvement rule, which forces new asks (bids) to be lower (higher) than those currently available in the market. In the experiments the price increment set at one cent. After a contract is completed all bids and asks are voided and new price quotes were accepted from interested participants.

Experimental Design

Experiments were conducted where agents participate in a circular flow economy. Each agent was endowed with money and one of the two goods in the economy. Agents could sell their endowment goods via a DA as described above. If an agent sold a good, they forewent the value of that good, but they received an amount of money equal

\(^3\) See Smith and Williams (1991) for a comparison of various DA rules including the one used in this study.
to the transaction price. In turn, subjects used their money to purchase units of the other good, for which the agents valued, via another simultaneously operating DA. Values and costs were induced using standard experimental procedures.

These laboratory economies consisted of ten subjects, five on each side of the two markets. In the experiments, the two goods markets were referred to as markets A and B. Hence, five subjects could buy good A and sell good B. The five remaining subjects were in the complementary role. Figure 1 shows the induced supply and demand conditions for the two identical markets. From this figure the equilibrium price of $1.50 and quantity of 15 units is readily observable. Subjects were assigned steps on these curves such that each participant’s share of the equilibrium surplus was $0.75 in each market.

In each experiment subjects participated in multiple trading periods. Each period a subject could trade up to four units of each good. The DA process continued until a trading period’s time limit of 240 seconds expired. Subjects were not endowed with money at the start of every trading opportunity but rather were required to carry money balances between some trading periods. The number of periods for which money was to be carried forward was common knowledge among the subjects. A subject earned a payoff equal to the change in the wealth of the agent. Thus at the end of trading a subject’s payoff was in part calculated as the sum of the induced values of goods purchased minus the induced cost, the foregone value, of goods sold. These gains from exchange were added to the difference between the subject’s ending balance and his initial endowment of money. As the value of the initial endowment of money would represent a gift from the experimenter, deducting this amount from the payoffs makes all
payoffs realizations of the gains from exchange. Since the real prices of the two goods are 150, any other price represents a transfer among market participants. Therefore, in equilibrium subjects should earn $1.50 per trading period, their share of the total economic surplus, $0.75 in each of the two markets.

These experiments were conducted at the Economic Science Laboratory. All subjects were undergraduate students at the University of Arizona and paid a $15.00 show up fee. After finishing the instructions, subjects participated in a series of paid practice markets\(^4\). These practice periods included some single market economies. The results presented in this paper consist of twenty trading periods for each session in which both markets were in operation. In one experiment, denoted SB, money circulated for only two periods each time it was introduced. In a second experiment, denoted LB, subjects had to maintain their money balances for a sequence of horizons that varied between two and eight trading periods. Between trading periods subjects received summary information about their activity in the previous period including the cost of goods sold, the value of goods purchased, transaction prices, money balances, and profit.

**Experimental Results\(^5\)**

The achieved levels of surplus in these economies are extremely high. In the last 5 trading periods (25% of the experiment) the total efficiency of the economy was at least 98% in both sessions. Further, total efficiency for a single trading period fell below 89% only once in session SB and never below this level in session LB. The overall average trading period efficiency was 96% in both sessions. Figures 2 and 3 plot average

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\(^4\) Subjects were recruited for four hours over two days. Both days began with the subjects reading the computerized directions, which are available from the author upon request. The paid practice periods occurred on day 1 and the experiment was conducted on the second day.

\(^5\) The data reported here are previously reported in Deck.
efficiency by period and give the number of contracts in each market for sessions SB and LB respectively.

Even in simple single market situations the double auction does not achieve the competitive outcome immediately, but rather reaches the competitive level after multiple replications. A multiple market economy following the circular flow pattern is significantly more complex than a single good market; hence, a priori one would expect relatively slow convergence in this setting. Therefore, these experiments were repeated for a relatively large number of trading periods, a total of twenty replications in each session. Also, individual subjects retained the exact same induced supply and demand schedules each trading period for the purpose of accelerating convergence. From Figures 2 and 3 these economies appear to quickly approach the equilibrium level of trade.

Econometric analysis, conducted to determine if the economies were converging to full efficiency, estimated \( \text{Eff}_t = \alpha + \beta \text{Eff}_{t-1} + \epsilon_t \) where \( \text{Eff}_t \) denotes the total efficiency of the economy at time \( t \) and \( \epsilon_t \) is an iid \( N(0, \sigma^2) \) shock. The steady state level of efficiency, \( \alpha/(1-\beta) \), is estimated to be 96%. To look for convergence to full economic efficiency a likelihood ratio test was conducted based on the restriction \( \alpha/(1-\beta)=1 \). As indicated in Table 1, this hypothesis is rejected at all standard levels of significance as the likelihood ration test statistic is \( \approx 17.7 \). A preliminary check of the coefficient on \( \beta \) indicates that autocorrelation is unlikely. This is confirmed by the Modified Breusch Godfrey Test, which has a p-value of \( \approx 0.87 \).

Besides the high level of achieved efficiency, the DA typically results in nearly uniform prices equal to the competitive level. Markets in this study are characterized by a single zero profit unit that should trade at the equilibrium price, unit 15 in Figure 1. A
strict measure of price performance would be that all trades should occur at 150. To relax this stringent measure for achieving competitive prices, a pseudo-price tunnel is constructed by removing the single zero profit unit from the market and using the resulting competitive price tunnel, a range of prices that support a competitive equilibrium. Figures 4-7 show nominal contract prices by market, displayed sequentially across an entire experiment. Marked on these figures are the competitive price and the pseudo-price tunnel. The basic price results are readily apparent from this chart. First, prices tend to center around the competitive prediction of 150. Second, prices are less volatile in later trading periods than in the initial phase, but price variance is not eliminated with market replications. By the later periods most of the contracts do occur at prices inside the pseudo tunnel, within 5 cents of the competitive price. However, 15% of the 225 trades that occur in the last four trading periods do so at prices outside this tunnel, one 33.33% above the theoretical benchmark. Additionally, in both sessions, prices in one market tend to remain above equilibrium while prices in the other market are below the equilibrium level on average. Unlike Williams et al. where budget constraints may explain systematic deviations from equilibrium pricing, the cause of this slippage is unclear. One potential explanation is that some subjects are slower to improve their prices, and that the complexity of this environment allows them to maintain this favorable asymmetry.

Figures 8-11 give the mean and the 95% confidence interval for transaction prices by market for each trading period in the last half of the experiment. The markets in Figures 9 and 10 show small variances in contract prices by the end of the experiment. However, of these two markets, only in market A of session LB does the competitive
price lay inside the confidence intervals. The other market for which the 95% confidence intervals contain the competitive price is market B in session LB because this session has such a large variance in prices. Interestingly, the market that has the tightest concentration of prices in the last three periods, market B in session SB, had extremely large variability in prices in periods 16 and 17, see Figure 9.

To formally examine the convergence of market prices towards the competitive prediction, the technique of Smith and Williams is employed. $\alpha^2(t)$ is defined as the average squared difference between nominal contract prices and the competitive equilibrium price in period $t$. Due to the existence of multiple markets in this experiment, this average was taken across markets thereby giving a sense of how close prices are to the theoretical level for the economy as a whole. Price convergence is revealed through ordinary least squares estimation, given in Table 2, of $ln(\alpha(t)) = a+bt+cS$ where $S$ is an indicator variable that is one for session SB. The results do show a significant exponential convergence rate of 4% per period. While prices are converging, it should be noted that the rate of convergence is only half the slowest rate reported in Smith and Williams. Table 2 also shows that there is no difference in price convergence between the two experiments at the 99% confidence level.

Conclusions

From these experiments one can conclude that in a circular flow economy the double auction achieves high levels of efficiency with prices tending to center near the competitive level. The efficiency properties of the DA that have been well documented in single market settings continue to hold in this more complicated situation. The overall average market efficiency for an experimental economy in this study was 96%, a figure
similar to that found in previous single market studies. However, econometrically these economies are found to not converge to full efficiency with replication.

With respect to prices, the DA’s performance is not as successful in a circular flow economy as in a single market. Relatively large price volatility remains in the markets even after twenty replications. However, prices are found to be converging towards the equilibrium prediction just a relatively slow rate. The circular flow design is fundamentally different from a single commodity experiment with respect to pricing. In a single market, buying at a price above value or selling below cost can never be optimal for an agent. However, with multiple commodities all agents can be identically well off under various realized price sequences. For example in this economy, each individual receives the same payoff from buying and selling three units of each good at the same price regardless of the specific price. So an agent may optimally sell below cost or buy above value if it allows that agent to complete a trade in the other market such that the total combined value of the two trades is positive. Therefore, it is not surprising that prices tend to be more volatile in a circular flow environment.

Acknowledgements

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References


Figure 1. Induced Supply and Demand

Price of A or B Commodity in Cents

Quantity of Commodity

Private Supply
Private Demand
Figure 1.
Double Auction Performance in a Circular Flow Economy
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Figure 2. Average Market Efficiency Across Trading Periods in Session SB

Solid lines separating consecutive trading periods indicate that money was reinitialized between the periods. The (A, B) pairs give the number of contracts that were completed in Markets A and B respectively.
Figure 2.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Solid lines separating consecutive trading periods indicates that money was reinitialized between the periods. The (A,B) pairs give the number of contracts that were completed in Markets A and B respectively.
Figure 3.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 4. Market A Contract Prices Session SB

Solid lines separating consecutive trading periods indicate that money was reinitialized between the periods. Numbers given on the chart denote contract prices occurring outside the range graphed. $\bar{p}$, $\bar{q}$, and $\bar{r}$ indicate the competitive equilibrium and the upper and lower range of the pseudo tunnel respectively. Contracts are given as the number of transactions with prices in the pseudo tunnel out of the total number of contracts for that trading period.
Figure 4.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 5. Market B Contract Prices Session SB

Trading Periods

Solid lines separating consecutive trading periods indicate that money was reinitialized between the periods. Numbers given on the chart denote contract prices occurring outside the range graphed. $\Omega$, $\eta$, and $\tilde{\eta}$ indicate the competitive equilibrium and the upper and lower range of the pseudo tunnel respectively. Contracts are given as the number of transactions with prices in the pseudo tunnel out of the total number of contracts for that trading period.
Figure 5.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 6. Market A Contract Prices Session LB

Trading Periods

Solid lines separating consecutive trading periods indicate that money was reinitialized between the periods. Numbers given on the chart denote contract prices occurring outside the range graphed. $\nearrow$, $\swarrow$ and $\searrow$ indicate the competitive equilibrium and the upper and lower range of the pseudo tunnel respectively. Contracts are given as the number of transactions with prices in the pseudo tunnel out of the total number of contracts for that trading period.
Figure 6.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 7. Market B Contract Prices Session LB

Solid lines separating consecutive trading periods indicate that money was reinitialized between the periods. Numbers given on the chart denote contract prices occurring outside the range graphed. $\sigma$, $\lambda$, and $\nu$ indicate the competitive equilibrium and the upper and lower range of the pseudo tunnel respectively. Contracts are given as the number of transactions with prices in the pseudo tunnel out of the total number of contracts for that trading period.
Figure 7.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 8. Confidence Intervals for Mean Transaction Prices Session SB Market A
Figure 8.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 9. Confidence Intervals for Mean Transaction Prices Session SB Market B
Figure 9.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 10. Confidence Intervals for Mean Transaction Prices Session LB Market A
Figure 10.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
Figure 11. Confidence Intervals for Mean Transaction Prices Session LB Market B
Figure 11.
Double Auction Performance in a Circular Flow Economy
Cary A. Deck
<table>
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<th>Estimate</th>
<th>T-statistic</th>
<th>p-value</th>
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<td>$\beta$</td>
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Convergence to Full Efficiency

Null Hypothesis: $\alpha/(1-\beta)=1$

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<td>0.0447</td>
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Modified Breusch Godfrey Test: $p$-value = 0.866272
Table 1.
Double Auction Performance in a Circular Flow Economy
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Table 2. Econometric Estimation of Price Convergence

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<tr>
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<tr>
<td>C</td>
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Model: $\ln(\alpha(t)) = a + bt + cS + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2)$
Table 2.
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