Analyzing the Impact of Mergers Using a Spatial Model*

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Abstract:
The study of mergers typically focuses on how market consolidation affects firm power or the cost structures of the producers. This paper applies a spatial model to merger analysis and demonstrates that the effects of mergers can include non-price consumer costs not present in other types of merger analysis. In this model three outlets compete based on price and distance in a Hotelling styled environment where firms can merge thereby allowing a single firm to own multiple retail outlets. A firm can operate or shutter any of its existing outlets; however, the locations of retail outlets are assumed to be exogenously determined. The results are that when a single firm owns two outlets, this firm will always find it more profitable to shutter an outlet, generating potentially substantial reductions in consumer surplus. Also developed is an extension of the model that allows for the existence of a potential entrant. The threat of entry is found to be sufficient to prevent shuttering by an incumbent firm with disjoint market segments.

JEL Classifications: D4, L1, L4
Keywords: Mergers, Spatial Model

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Introduction

In the classic model of spatial competition, due to Hotelling (1929), agents compete along a one-dimensional bounded segment with uniformly distributed consumers with a linear travel cost. Braid (1991) adapts the traditional one-dimensional problem to higher order settings investigating competition over squares and diamonds, which demonstrates the spatial model’s applicability to a geographic market. The uniform density assumption of the classic model is relaxed by Anderson, Goeree, and Ramer (1997) further serving to improve the model’s applicability to locational competition. Spatial models can also be used to study product differentiation in attribute space. In this context, distance represents a customer’s inability to achieve their preferred level of a product characteristic. Irmen and Jacques-Francois (1998) use a spatial model to show why firms might differentiate along a dominant characteristic but not along other dimensions. Cremer and Thisse (1991) show that a large class of Hotelling type models is equivalent to vertical differentiation. Other researchers studying product differentiation include Vandenbosch and Weinberg (1995) and Veendorp and Majeed (1995). As a whole, this body of the industrial organization literature demonstrates the breadth of applications for spatial models.

This versatile tool can also yield insight for those studying the effects of horizontal or vertical mergers. Heywood, Monaco, and Rothschild (2001) apply a spatial model to merger analysis. However, in their framework only adjacent firms are allowed to merge and the firm is forced to operate each outlet. This research extends the application of spatial models to analyze mergers where the firms have the ability to shutter outlets and to merge with outlets serving a nonbordering segment of the market. This work also addresses the effects a potential entrant has on the behavior of merged firms. The next section develops the model of spatial competition. The following section discusses the general results. A separate section provides an extension of
the basic model for examining the impact of potential entry. A final section contains concluding remarks.

**Model**

Firms are assumed to produce a perfectly substitutable good and compete based on price over the geographic distribution of consumers. Firms can operate multiple retail outlets; however, the location of all retail outlets is taken to be exogenous. This assumption is appropriate since a merger typically involves incumbent firms. Further, each firm has the option of closing any of the retail outlets under its control. Consumers, who are assumed to have a perfectly inelastic demand for one unit of the good, are distributed uniformly over a closed bounded line segment, [0,1]. These consumers are assumed to face a quadratic cost associated with traveling. A consumer located at \(x\) would prefer to purchase one unit from a retail outlet located at point \(a\) that charges a price \(P_a\) rather than purchasing from another outlet located at point \(b\) that charges a price \(P_b\) if \(P_a + (x-a)^2 < P_{a} + (x-b)^2\). The consumer will ultimately complete the purchase from the outlet offering the lowest total expense, price plus travel cost. If multiple outlets offer the lowest total cost to a consumer then the consumer randomly selects one of these low cost outlets from which to purchase.

A firm operating outlets indexed by \(i\) seeks to maximize its profit, \(\pi\), which is given by equation (1).

\[
(1) \quad p = \sum_{i=1}^{n} P_i q_i - \text{cost}(q_1, \ldots, q_n).
\]

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1 As discussed in the previous section, this assumption can be replaced with accurate population data when applying the model to a specific merger. The assumption of a uniform distribution of customers is purely for simplicity of exposition.
Costs are also a function of \( n \), the number of retail outlets a firm operates, and of \( q_1, \ldots, q_n \), the quantity sold at each outlet. Total costs are assumed to be Transylvanian having the form given in equation (2).

\[
(2) \quad \text{cost}(q_1, \ldots, q_n) = F + nf + k \sum_{i=1}^{n} q_i.
\]

\( F \) is a fixed cost associated with operating a firm in this industry, while \( f \) is the fixed cost associated with operating a retail outlet. The marginal cost of production is assumed to be constant at a per unit cost of \( k \) and \( q_i \) is the quantity sold at retail outlet \( i \). This cost function captures the notions of synergy and economies of scale that are generally argued to be present in mergers.\(^2\) Baumol, Panzar, and Willig (1982), show that such Transylvanian cost structures can yield natural monopolies in many environments; however, that is not the case in spatial models. A natural monopoly is a situation where one large firm incurs cost savings that allow the firm to undercut all competitors. Since, in the spatial model not all cost are internal to the firm, namely consumer travel costs, one firm may not be able to undercut other firms’ total cost to consumers even with the economies of scale present. That is consumers may prefer an outlet with a higher retail price and lower travel cost to a low price outlet with a high travel cost.

The maximization problem of a firm operating a single outlet can be described in \( \mathbb{R}^2 \) as shown in Figure 1. Since \( F \) and \( f \) are fixed costs, these parameters do not affect the firm’s optimization problem. Centered about a retail outlet’s location is a parabola, the vertical height of which represents the total cost a consumer located at that point would expend if they choose to purchase from that particular outlet. The firm operating an outlet sets a price, which forms the minimum of the parabola. An outlet’s demand is the subinterval of \([0,1]\) where its parabola lies

\( ^2 \) All results remain fundamentally unchanged if the cost structure is relaxed to \( \text{cost}(q_1, \ldots, q_n) = f(n) + k \sum q_i \) where \( f(\cdot) \) is a nonnegative, weakly increasing function of \( n \).
below the parabola associated with each of the other outlets. The operating profit of the outlet is equal to the area of a rectangle with a height equal to the minimum of the parabola minus the average variable cost and a width equal to the segment of the market over which the outlet’s parabola is below that of all rival outlets including any outlets owned by the same firm. Due to the quadratic nature of travel costs a retail outlet’s demand is a continuously differentiable function of its own price.

**Competition Analysis**

Attention is restricted to competition among three firms each operating one retail outlet, the simplest situation in which two firms can merge and not result in a monopoly. Let the three independently owned outlets A, B and C be located at a, b and c respectively with 0 < a < b < c < 1. In this set up there are two potential sorts of mergers; mergers of firms with no contiguous consumers (a merger of firm A & C) and mergers of firms with adjacent sales areas (a merger of A & B or B & C). In the remainder of this paper a firm is referred to by the outlets under its control. For example, if firm A and firm C merge the resulting firm is denoted as firm AC. The focus of the paper first turns to three firm competition, which serves as a baseline for comparison with the two merger cases that are discussed separately.

**Three Firm Competition**

In this competitive case prices are simultaneously and set at each retail outlet to maximize the owning firm’s operating profits. When each retail outlet is owned independently, in equilibrium every firm will have positive operating profits. Suppose that some firm, located at y, earned zero profit, then the firm could set a price of ε+k > k where ε is sufficiently small such
that every person within a $\delta$ neighborhood of $y$ would prefer outlet $y$, resulting in a operating profit of $\varepsilon \delta > 0$. This follows since 1) no competing firm will set a price below $k$, as this would generate operating losses while an infinite price would guarantee non-negative operating profits and 2) the location of each outlet is unique.

The sales boundary between neighboring outlets is determined by the consumer located at $x$ who is indifferent between the total costs of purchasing from the two neighboring outlets. For example the boundary between A and B is $x$ such that $(x-a)^2 + P_a = (x-b)^2 + P_b$. Therefore the boundary is $x = (b^2 - a^2 + P_b - P_a)(2(b-a))^{-1}$. As outlet A will serve every customer between 0 and the this boundary, firms A’s demand is given by equation (3). The demands faced by firms B and C are similarly calculated and are given by equations (4) and (5) respectively.

\begin{align*}
(3) & \quad (b^2 - a^2 + P_b - P_a)[2(b-a)]^{-1} \\
(4) & \quad (c^2 - b^2 + P_c - P_b)[2(c-b)]^{-1} - (b^2 - a^2 + P_b - P_a)[2(b-a)]^{-1} \\
(5) & \quad 1 - (c^2 - b^2 + P_c - P_b)[2(c-b)]^{-1}
\end{align*}

The resulting first order conditions for profit maximization\(^3\) for each of the firms A, B, and C are given by (6), (7), and (8) respectively.

\begin{align*}
(6) & \quad (b^2 - a^2 + P_b - 2P_a + k)[2(b-a)]^{-1} = 0 \\
(7) & \quad (c^2 - b^2 + P_c - 2P_b + k)[2(c-b)]^{-1} - (b^2 - a^2 + 2P_b - P_a - k)[2(b-a)]^{-1} = 0 \\
(8) & \quad 1 - (c^2 - b^2 + 2P_c - P_b - k)[2(c-b)]^{-1} = 0
\end{align*}

The prices and profits of each firm are presented in Table 1. The fact that these price solutions are global maxima is readily verifiable from the second order conditions given in equation (9), (10), and (11) for firms A, B and C respectively given the location order is $a < b < c$.

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\(^3\) These F.O.C., equations (6), (7), and (8), are derived from optimizing equation (1) after substituting in equation (2) for the cost function and using the appropriate firm demands, equations (3), (4), and (5), for firms A, B and C respectively. Similar first order conditions are used to determine the other prices and profits discussed in this paper, but the individual steps are omitted for brevity.
Non-Contiguous Merger: A Merger Between Firms A and C

Firm AC, the firm formed by the merger of firms A and C, has to decide if it should operate both available retail outlets before the simultaneous price setting game occurs. If firm AC decides to operate both outlets in the post-merger environment, the first order conditions for maximizing firm profits remain unchanged for all three retail outlets. Therefore, the firm’s profits equal the sum of profits for the two outlets prior to the merger plus F and the costs born by consumers remain constant. However, a firm has two choices of outlets to close and a firm will always find it more profitable to run only one of the outlets, that is

$$\Pi_{AC(a,c)} < \max\{\Pi_{AC(a)}, \Pi_{AC(c)}\}.$$  

Table 2 lists the profit information relevant to a firm making this decision. For some parameter values the firm may be indifferent as to which outlet to shut down, the simplest case of this is when the three outlets are symmetric about the midpoint of the line segment, that is \((a,b,c) = (\cdot5-\cdot\epsilon, \cdot5, \cdot5+\cdot\epsilon)\).

Consumers can be adversely affected by this type of merger in two ways; increased travel cost and higher prices. The measure of the impact on consumers is the change in the area below all of the outlet parabolas over the interval \([0,1]\) for the outlets in operation before and after the merger. This yields the reduction in consumer surplus caused by the merger. While, a portion of this reduction in consumer surplus is transferred to producers through higher prices, some of the loss in surplus is due to reduced efficiency with the increase in travel cost. Not only do

\[\begin{align*}
(9) & \quad -(b-a)^{-1} < 0 \\
(10) & \quad -(c-b)^{-1} - (b-a)^{-1} < 0 \\
(11) & \quad -(c-b)^{-1} < 0
\end{align*}\]

\[4\text{ In this notation the first subscript identifies the firm based on the outlets under its control while letters in parenthesis identify which outlets are in operation in the market.}\]
consumers who previously shopped at the closed outlet face a change in travel costs, but as the boundaries between sales areas of remaining outlets change so do the travel costs of consumers located along that frontier. Without loss of generality assume $\Pi_{AC(a)} > \Pi_{AC(c)}$ so that firm AC will close the outlet located at c and operate the outlet located at a. The resulting change in market efficiency due to the merger is given in (12).

(12) $F+f-([144(c-a)]^{-1}(7a^4-32a^3+16a^2+4a^2b-14a^2c^2+28a^2c+112ab+20cb^2a+112c^2a-
80b^2a-20c^2ba-20c^3a-144ca+20b^3a+27c^4-108c^3+108c^2+20c^3b-20b^2+80b^2c-20b^2c^2-
72bc-20cb^3-4c^2b)).$

Antitrust authorities could eliminate the adverse effects of this type of merger by simply requiring the newly merged firm to operate both outlets. The firm would still realize profits greater than the sum of the profits of the realized when the two outlets were independently owned due to the cost reduction $F$, and the merged firm will set outlet prices identical to the pre-merger outlet prices. Hence, allowing the merger and implementing such a remedy would actually increase the overall level of efficiency in the market.

**Contiguous Merger: A Merger Between Firms A and B**

Again, the newly merged firm must make a shuttering decision prior to the simultaneous price setting game. Based on the profit calculations presented in Table 3, firm AB would always maximize profits by operating only outlet a. Formally, $\Pi_{AB(a)} > \max(\Pi_{AB(a,b)}, \Pi_{AB(b)})$. This result is counterintuitive as the firm AB has the ability to set prices at location a so as to exploit consumers who are isolated from the competitive frontier between outlets b and c. However,

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5 Software that numerically analyzes the relevant profit information is available from the author.
6 By symmetry, analysis of a merger between firms B and C is identical to the merger between firms A and B and is hence omitted.
7 Software that numerically analyzes the relevant profit information is available from the author.
profits from the reduction in competition outweigh any exploitative gain. As before, the welfare effects can be calculated by comparing changes in travel costs and firm profits. In the contiguous merger situation, consumer surplus would decrease and the resulting change in market efficiency is given in (13).

(13) \[ F+f-\left\{\frac{144(c-a)}{(c-a)^2}\right\}^{1/2}(26a^2c^2-20c^3b-20ca^2b+40cb^2a+16a^2b+7c^4+7a^4-20b^2-20c^2-4c^2b+32b^2a-32b^2c+28a^2c+4a^2b-16ca+56bc-16ab+4c^3-20a^2b^2-32a^3-20c^2b^2+20c^3b-20c^3a+20a^3b-20a^3c)\]  

Unlike the case of a disjoint merger, forcing firm AB to operate both retail outlets will not eliminate all the adverse effects of the merger. Instead, the merged firm would behave in the intuitive way setting \( P_A = (b^2-a^2)/2 + P_B \). Prices at all three outlets would be higher than the pre-merger environment and consumer surplus would remain below the three firm competitive baseline. These findings are consistent with the results described in Heywood, Monaco, and Rothschild (2001) for the corner case in an N-firm environment.

**Potential Entry**

A new entrant will compete in a market if the firm can attain sufficient profits. Therefore, if a merger results in super-competitive profits for incumbent firms then a new competitor maybe likely to enter the market. Such a potential entrant could be expected to affect the behavior of incumbent firms in the post-merger environment. In the non-contiguous merger case analyzed in the previous section, antitrust regulation can maintain the competitive outcome. However, the threat of entry is also sufficient to sustain three outlet competition after the merger of two non-contiguous firms, thereby eliminating the need for such regulation. Analysis of the
contiguous case depends upon the location of the incumbent firms’ outlets and requires a case-by-case investigation, which is not presented in this paper.

In the spatial model developed in this paper, the locations of incumbent firms are taken as given. As stated above, this assumption is appropriate as the current market environment provides a frame of reference for evaluating the impact of a merger. However, a new firm, firm D, would necessarily need to select a location. The potential entrant is allowed to choose both a location, $d \in [0,1]$ s.t. $d \notin \{a,b,c\}$, and set a price, $P_d$. Assuming the pre-merger competition environment represents the status quo, $F+f$, the fixed cost of entry, must be sufficiently high such that for any possible choice of location selected by the potential entrant profits would not offset these costs if the new entrant were to face competition from three independent firms.

A more complicated sequential game is required to analyze the impact that a potential new entrant has on the post merger market. This game is represented in Figure 2 for the case where firm AC would prefer to shutter outlet c in the absence of the new entrant. There is a symmetric case where firm AC would chose to operate outlet c only. In the event of a merger, the post-merger incumbents first decide to either shutter or operate each outlet simultaneously. Conditional on the decision of the incumbents the potential entrant decides to either not enter the market or to enter at location d subject to the aforementioned constraints. In the final stage all firms operating in the market set retail outlet prices simultaneously.

Immediately two features of this game are apparent. Due to the assumption that $F+f$ is sufficiently high so that firm D could not enter the premerger market profitably, if the three outlets at a, b, and c are operating, none of the outcomes where all four outlets operate simultaneously can be an equilibrium. Hence, if firm AC operates both outlets a and c then firm D will not enter the market. The second prominent feature pertains to the situation in which
outlet c is shuttered. In this case firm d has the ability to open an outlet at \( d = c + \varepsilon \) where \( \varepsilon \) is sufficiently close to 0 such that \( \Pi_{D(d=c+\varepsilon)} \approx \text{premerger } \Pi_{C(c)} \). As \( F+f \) was sufficiently low so as to allow firm C to earn a profit before the merger, then selecting \( d \) sufficiently close to \( c \) should generate positive profits for firm D in the post merger environment. Therefore, firm D not opening an outlet cannot be an equilibrium in the subgame in which firm AC shutters outlet c.

In the scenario, that firm AC shutters outlet c, firm D could choose to operate in one of three regions; \([0,a), (a,b), \) or \((b,1]\). Which region firm D would select depends on the specific values of \( a \) and \( b \). However, using the information presented in Table 4, it follows that Firm AC’s profit from operating outlet a alone and facing competition from B and D is strictly less than its profit from operating both a and c in order to prevent entry by firm D. This result also follows intuitively. Assume that firm AC closes the outlet located at c. If \( d \) is chosen to be greater than \( b \), firm AC is essentially in the position of firm A in the pre-merger environment. In essence, by shutting down the outlet at c firm AC basically gives away the profits outlet c could have earned. Should D select to open at a location less than \( b \), then firm AC finds itself in a more competitive environment then firm A experienced prior to the merger and hence will achieve smaller profits than originally attained by firm A.

Firm AC will optimally select to operate both outlets a and c. The reduction in profits from internal competition is smaller than the loss in profits from an additional outside competitor. This does not mean that firm A and firm C would no longer find it optimal to merge as the combined firm operating two outlets does receive a cost savings. Hence, in the presence of a potential entrant, the market will remain as competitive as and more efficient than the pre-merger baseline without any formal remedy.
Conclusions

The general results of this model indicate that in a three outlet environment whenever firms merge “shuttering” will occur. This has the effect of reducing consumer surplus by increasing travel costs. The overall effect on efficiency depends on the magnitude of the avoided fixed costs. Also, firms will receive a transfer of surplus from consumers through higher prices. The impact of antitrust regulation depends on the spatial relation of the merging firms. If the two firms do not operate neighboring sales areas then under a non-shuttering remedy one would expect to see prices remain unchanged post merger. However, when a firm has retail outlets that serve adjacent sales areas, prices will remain higher and consumer surplus will remain lower even under such regulation. A firm operating in an insulated market segment will set prices at its competitive outlet and then charge a convenience premium at the noncompetitive outlet.

The threat of another firm entering the market is sufficient to force firms in non-contiguous mergers to behave competitively. In this case the merger does not adversely affect consumers as all three outlets remain in operation. As travel costs and prices remain unchanged there is no loss in consumer surplus. Firm profits increase for the merged firm, but not through increased transfers from buyers, but rather by efficiency gains from the reduction of the average cost of production.

This type of analysis provides a useful tool for examining the implications of a proposed merger. Current FTC guidelines base the likelihood of the exercise of market power on the Herfindahl-Hirschman Index.\(^8\) Unlike investigations relying on HHI calculations or other general measures, spatial analysis provides a direct measure of a merger’s impact upon consumers for many cases subject to antitrust scrutiny. The model has vast applicability due to

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\(^8\) The FTC’s guidelines for horizontal mergers are available online at http://www.ftc.gov/bc/docs/horizmer.htm.
its considerable flexibility including expansion to multi-dimensional space for geography and product characteristics, adjustment for accurate population density data as well as production and travel cost structures.

References


Table 1. Prices and Profits for Firms A, B, and C in Independent Competition

<table>
<thead>
<tr>
<th>Firm</th>
<th>Price ( P_a )</th>
<th>Profit ( \Pi_a )</th>
<th>Price ( P_b )</th>
<th>Profit ( \Pi_b )</th>
<th>Price ( P_c )</th>
<th>Profit ( \Pi_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[6(a-c)]^{-1} (2b^2a - 2b^2c - 3a^3 + 2a^2c + 2b^2c + c^2a + a^2b + 2ca - 2bc - 2ab) + k]</td>
<td>[72(a-c)^2]^{-1} (3a^2 + 2ab - 2ca - c^2 + 2b - 2c - 2bc)^2 (b-a) - f - F]</td>
<td>[3(a-c)]^{-1} (2b^2 - c^2b + c^2a - b^2a + b^2c - a^2c + a^2b + 2ca - 2bc - 2ab) + k]</td>
<td>[18(a-c)]^{-1} (a^2b - a^2c + 2ca - b^2a + c^2a - 2ab + 2b^2c - c^2b - 2bc) (c-a+2) - f - F]</td>
<td>[6(a-c)]^{-1} (8ca - 6c^2 - 8ab + 4bc - 2c^2a + 2c^3 + 2b^2a - 2b^2c + 2b^2c - c^2b - a^2c + a^2b) + k]</td>
<td>[72(a-c)^2]^{-1} (2b - 2bc + 2ab - 8a + 6c - 3c^2 + a^2 + 2ca)^2 (c-b) - f - F]</td>
</tr>
</tbody>
</table>
Table 2. Prices and Profits for Merged Firm AC Competing Against Firm B

<table>
<thead>
<tr>
<th>Firm AC operating both outlets a and c</th>
<th>Price Pa</th>
<th>[6(a-c)]^{-1}(2b^2a-2b^2c-3a^3+2a^2c+2b^2c^2b+c^2a+a^2b+2ca-2bc-2ab)+k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit Pa</td>
<td>[72(a-c)^2]^{-1}(3a^2+2ab-2ca-c^2+2b-2c-2bc)^2(b-a) - f - F/2</td>
<td></td>
</tr>
<tr>
<td>Price Pc</td>
<td>[6(a-c)]^{-1}(8c^2-2c+3a^2+2b^2c^2b+c^2a-a^2c+a^2b)+k</td>
<td></td>
</tr>
<tr>
<td>Profit Pc</td>
<td>[72(a-c)^2]^{-1}(2b-2bc+2ab-8a+6c-3c^2+a^2+2ca)^2(c-b) - f - F/2</td>
<td></td>
</tr>
<tr>
<td>Firm AC operating outlet a only</td>
<td>Price Pa</td>
<td>((3)^{-1}(2b-2a+b^2-a^2)+k)</td>
</tr>
<tr>
<td>Profit Pa</td>
<td>((18)^{-1}(a+2+b)(b^2-a^2+2b-2a) - f - F)</td>
<td></td>
</tr>
<tr>
<td>Firm AC operating outlet c only</td>
<td>Price Pc</td>
<td>((3)^{-1}(4c-4b-c^2+b^2)+k)</td>
</tr>
<tr>
<td>Profit Pc</td>
<td>((18)^{-1}(b-4+c)(c^2-b^2-4c+4b) - f - F)</td>
<td></td>
</tr>
</tbody>
</table>

The profit to Firm AC from operating both outlets simultaneously is the sum of profits at the individual outlets.
Table 3. Prices and Profits for Merged Firm AB Competing Against Firm C

<table>
<thead>
<tr>
<th></th>
<th>Price ( P_a )</th>
<th>Profit ( \Pi_a )</th>
<th>Price ( P_b )</th>
<th>Profit ( \Pi_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm AB operating both outlets a and b</td>
<td>((6)^{-1}(b^2-3a^2+4c-4b+2c^2)+k)</td>
<td>((24)^{-1}(b^2-3a^2+4c-4b+2c^2)(a+b) - f - F/2)</td>
<td>((3)^{-1}(2c-2b+c^2-b^2)+k)</td>
<td>((36)^{-1}(4-3a-b+2c)(2c-2b+c^2-b^2) - f - F/2)</td>
</tr>
<tr>
<td>Firm AB operating outlet a only</td>
<td>((3)^{-1}(2c-2a+c^2-a^2)+k)</td>
<td>((18)^{-1}(a+2+c)(c^2-a^2+2c-2a) - f - F)</td>
<td>((3)^{-1}(2c-2b+c^2-b^2)+k)</td>
<td>((18)^{-1}(b+2+c)(c^2-b^2+2c-2b) - f - F)</td>
</tr>
</tbody>
</table>

The profit to Firm AB from operating both outlets simultaneously is the sum of profits at the individual outlets.
Table 4. Profits for Merged Firm AC Competing Against Firms B and D

<table>
<thead>
<tr>
<th>Profits for Firm AC operating both outlets competing with Firm B</th>
<th>[72(a-c)^2]^{-1}[(3a^2+2ab-2ca-c^2+2b-2c-2bc)^2(b-a) + (2b-2bc+2ab-8a+6c-3c^2+a^2+2ca)^2(c-b)] -2f -F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits for Firm AC operating only outlet a competing with Firms B and D with d&lt;a</td>
<td>[18(d-b)^{-1}(d^2a-d^2b+2bd-a^2d+b^2d-2da+2a^2+a^2b-b^2a-2ab)(b-d+2) -f -F</td>
</tr>
<tr>
<td>Profits for Firm AC operating only outlet a competing with Firms B and D with a&lt;d&lt;b</td>
<td>[72(a-b)^{-1}(3a^2+2ad-2ba-b^2+2d-2b-2db)^2(d-a) - f -F</td>
</tr>
<tr>
<td>Profits for Firm AC operating only outlet a competing with Firms B and D with b&lt;d</td>
<td>[72(a-d)^{-1}(3a^2+2ab-2da-d^2+2b-2d-2bd)^2(b-a) - f -F</td>
</tr>
</tbody>
</table>
Figure 1. \( \mathbb{R}^2 \) Depiction of Three Firm Competition

\[ \begin{align*}
0 & \quad a & \quad b & \quad c & \quad 1 \\
\text{k} & \quad P_a & \quad \Pi_b & \quad Q_c
\end{align*} \]

a, b, and c indicate the locations of the three outlets along the segment over which customers are uniformly distributed. k is the marginal cost of production and \( P_i, \Pi_i, Q_i \) denote the price, profit and quantity associated with outlet i.
Figure 2. Sequence of Decisions in Environment with Potential Entrant

After Firms A and C merge into Firm AC

φ denotes the simultaneous price game described in the text. When in operation, location d is chosen by Firm D from within the constrained range so as to maximize profits in the pricing game in the final stage. All operating outlets are listed in the tree by location in increasing order.