Downside Consumption Risk and Expected Returns

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Abstract

Economists have long recognized that aversion to downside risk is an important feature of human behavior. Indirect empirical evidence also suggests that downside risk contributes to financial assets' risk premia. In this paper we test the restrictions on consumption and asset returns implied by a recursive utility which incorporates aversion to downside risk using two different models: Rank-Dependent Expected Utility and Disappointment Aversion. Both models nest the standard CCAPM with recursive utility and are estimated using a cross-section of asset returns. Our estimates indicate that downside risk models considerably improve upon the performance of CCAPM and fit the data similarly or better than a linear three factor model. The restrictions to the standard CCAPM are strongly rejected. A decomposition of risk premium indicates that the portion attributed to the downside risk varies systematically across test portfolios and contributes to value and size risk premia. Thus, the standard consumption-based channel is insufficient to characterize risk premium and a downside risk factor is required.

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1 Introduction

There exists considerable experimental evidence that accurate characterization of choice under risk calls for a utility function dependent on outcome's position relative to other potential outcomes or relative to a reference point.[1] Several well-known utility functions use such mechanisms, for example Disappointment Aversion utility (Gul, 1991), Generalized Disappointment Aversion (Routledge and Zin, 2010), Rank-Dependent Expected Utility (Quiggin, 1982; Yaari, 1987), Cumulative Prospect Theory (Tversky and Kahneman, 1992) and Reference-Dependent Utility (Koszegi and Rabin, 2006). In recent years there emerged a rapidly growing field in financial economics which develops theoretical models based on utilities with loss aversion, disappointment aversion and rank-dependency and applies them to standing problems in portfolio choice, asset pricing and corporate finance. The motivation for using such utility functions largely comes from experimental data and lacks direct non-experimental support. On the other hand, there is also a substantial empirical evidence in asset pricing which points to the importance of risk factors related to downside risk or, more broadly, state of the economy risk. These risk factors significantly improve the explanatory power of CAPM or Consumption CAPM (CCAPM). However, such factors are often justified on the introspective basis and the empirical tests typically do not correspond to any specific model of investor behavior.[2]

In this paper we test the cross-sectional restrictions on consumption and asset returns implied by two models with downside risk aversion: Rank-Dependent Expected Utility (RDEU) and Disappointment Aversion (DA). Our results establish an explicit link between downside risk and risk premia observed in financial assets. This accomplishes two goals. First, it demonstrates that the behavior observed in experimental settings is relevant for pricing real world securities. Thus, we provide essential non-experimental evidence to support theoretical models with aversion to downside risk. Second, we show that downside risk is an important and distinct factor which goes beyond the standard consumption-based channel for generating risk premium. This establishes a missing formal link between the observed features of investor risk preferences and empirical success of factors intended to capture downside risk.

[1] This idea appears as early as in Markowitz (1952) with an investor who evaluates losses relative to "customary wealth". For reviews of this extensive literature see Shoemaker (1982), Camerer (1995) and Starmer (2000).

[2] An exception is Epstein and Zin (2001) who test the ability of Disappointment Aversion model to explain market risk premium.

As the basis for our empirical investigation we use the recursive utility model of Epstein and Zin (1989). The model allows for separation of elasticity of intertemporal substitution from other parameters governing risk attitude in atemporal setting. The recursive utility allows one to use a variety of certainty equivalent functions which capture static risk preferences. In this paper we focus on two types of functions most relevant for the analysis of downside risk: Rank-Dependent Expected Utility (Quiggin 1982, and Yaari 1987) and Disappointment Aversion (Gul, 1991). Both utility functions nest the Expected Utility (EU) as a special case and can be empirically distinguished by a parameter restriction. The former model captures downside risk through the mechanism of decision weights. All possible outcomes are ranked from the worst to the best and assigned decision weights by transforming the original probability distribution. The weighting function can be selected to overweight some outcomes (e.g. unfavorable) relative to others (e.g. more favorable). We emphasize that this is not a model of subjective beliefs and the weights are intended to convey risk attitude towards probabilities, just as the expected utility function captures risk attitude towards levels of consumption.[3] The DA utility has a conceptually similar mechanism, it overweights unfavorable outcomes below certainty equivalent and underweights the outcomes above the certainty equivalent. The difference is that, in the case of DA, the overweighted outcomes are determined implicitly by the same parameter which determines the scale of overweighting. RDEU allows more flexibility in the choice of decision weighting function.

To estimate each model, we use intertemporal restrictions on consumption and asset returns implied by their corresponding stochastic discount factors (SDF). Each model is using an overweighting mechanism to capture downside risk aversion separately from the marginal utility of consumption. As a result, each SDF is a product of two parts. The first part is the same for both models and coincides with the standard EU-based SDF for recursive utility (Epstein and Zin, 1989). The second part of the SDF is different across the models and corresponds to the overweighting functions capturing downside risk. We show that this structure of the SDF implies that risk premium of an asset can be decomposed into three parts. First is the covariance with the standard EU-based SDF, second is the covariance with the downside risk factor and the third term captures the covariance of

[3]For this reason Yaari also refers to this model as "Dual Theory of Choice". According to RDEU, the decision maker is assumed to be unbiased and aware of the true probability distribution. Unlike subjective probabilities, the decision weights depend on the event's rank and may change through the actions of the decision maker. For example, what is a bad and what is a good outcome depends on whether one has a short or a long position in a particular asset. For an axiomatization of the RDEU see, for example, Abdellaoui (2002).

the two components of the SDF. We use this decomposition to evaluate the importance of downside risk.

We estimate both models by GMM using monthly aggregate consumption data, the returns on Fama-French portfolios sorted by size and book-to-market ratios, the t-bill return and the value weighted market portfolio return. For comparison we also estimate EU-based CCAPM with recursive utility (EU-CCAPM) and the linear three-factor Fama-French CAPM. In summary, our results indicate that downside risk feature considerably improves the performance of the models relative to EU-CCAPM. The restrictions of EU-CCAPM are strongly rejected. The downside risk models' overidentifying restrictions are not rejected and their overall fit is comparable to (based on the R^2 metric) or better than (based on the χ^2 distance) the Fama-French three factor model. The decomposition of risk premium shows that the component associated with downside risk varies systematically across portfolios and that value stocks and small stocks have larger exposure to downside risk than do growth stocks and large stocks, suggesting that the value and size premia are related, in part, to the downside risk. Robustness tests with industry portfolios, as advocated by Lewellen, Nagel and Shanken (2010), confirm our main findings. We also use factor-mimicking portfolios to compare the performance of downside risk SDFs and the Fama-French factors in the original and extended samples. We find that downside risk factors are always significant and that they either reduce or eliminate the significance of the Fama-French factors.

Our results contribute to the growing literature exploring the role of downside risk and non-expected utilities in portfolio choice and asset pricing. Several papers explore theoretical models based on RDEU and DA. Epstein and Zin (1990) consider an endowment model with RDEU to study the equity premium with the focus on first-order risk aversion. Ang, Beakert and Liu (2005) consider a portfolio choice model with DA utility and show that it may help explain why some investors avoid stocks. Routledge and Zin (2010) develop Generalized DA (GDA) model to allow for time-varying risk aversion by modeling flexible disappointment threshold. Unlike the original DA model, GDA can generate time-varying price of risk. Chapman and Polkovnichenko (2010) study the implications of preferences heterogeneity for a variety of non-expected utility models, including DA and RDEU. Our results contribute to this literature by providing non-experimental support for the importance of downside risk in the cross-section of risk premia. This indicates that other interesting implications of the models based on downside risk aversion may be present in the financial data.

There is also a number of related empirical papers which explore CAPM and CCAPM with various risk factors proxying for downside risk or state of the economy risk. This literature has a long tradition and dates as far back as Kraus and Litzenberger (1976) who show that skewness of portfolio return is related to its risk premium over and above the covariance with market-wide risk. The work by Harvey and Siddique (2000) shows that conditional co-skewness of asset and market returns is related to risk premium. Skewness is one of the statistics that characterizes asymmetry in the tails of a distribution and is related to downside risk. Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) apply respectively CAPM and CCAPM to show that certain additional factors which proxy for economic conditions ("bad events") improve the fit of the models. Ang, Chen and Xing (2005) measure the asymmetry in conditional portfolio beta during market downturns and show that it is related to portfolio risk premium over and above standard risk factors. Post and van Vliet (2006) explore SSD-efficiency of the market portfolio and demonstrate that higher order moments capturing asymmetry in the returns distribution due to downside risk make up for an apparent inefficiency under the standard mean-variance criterion. A common thread in this literature is that standard market or consumption risk factors are insufficient to characterize the cross-section of expected returns. However, the additional risk factors serve as proxies for downside risk and do not represent a test of any specific model of investor behavior. The main difference in the present paper is that we consider structural models of the SDF which explicitly confirm the link between downside risk and the cross section of returns. By rejecting the EU-CCAPM specification our tests directly demonstrate that downside risk contributes to risk premium in financial assets. Epstein and Zin (2001) appear to be the only other paper which considers structural estimation with DA utility. They show the importance of downside risk for market risk premium while this paper is focused on the cross sectional implications of DA and RDEU.

The rest of this paper is organized as follows. Section 2 develops CCAPM with RDEU and DA utility. Section 3 describes estimation methodology and data. Main estimation results are presented in section 4 and their economic intuition is discussed in section 5. Section 6 investigates robustness of the results and section 7 concludes.

2 Consumption CAPM with aversion to downside risk

As a point of departure for our investigation we use recursive utility specification formulated by Epstein and Zin (1989). The utility is represented as a recursion:

$$U_t = W(c_t, \mu_t(U_{t+1})) \tag{1}$$

where W is an aggregator function and μ_t is a certainty equivalent representing atemporal risk preferences. A convenient and widely used aggregator function is that of constant elasticity of substitution given by:

$$\begin{aligned} W(c,x) &= ((1-\beta)c^\rho + \beta x^\rho)^{\frac{1}{\rho}} \quad , \quad 0 \neq \rho < 1 \\ W(c,x) &= e^{(1-\beta)\log(c)+\beta\log(x)} \quad , \quad \rho = 0 \end{aligned} \tag{2}$$

where β is time preference parameter and ρ determines the elasticity of intertemporal substitution (EIS) as $\sigma = \frac{1}{1-\rho}$. When μ is given by EU, this utility function allows for separation of EIS and the risk aversion (Epstein-Zin recursive utility). More generally, the recursive utility allows us to use a variety of certainty equivalents for atemporal risk preferences while separating the EIS from other parameters. To explore the role of downside risk in expected returns we consider two well known certainty equivalents which allow us to model the additional emphasis a decision maker may place on unfavorable outcomes. We consider the Rank-Dependent Expected Utility developed independently by Quiggin 1982 and Yaari 1987, and the Disappointment Aversion utility developed by Gul (1991).

To define certainty equivalent for RDEU, consider an agent who evaluates risky consumption c. Let x be a random variable associated with states of consumption and let P denote its cdf. Assume that outcomes are ranked by x with higher values of x corresponding to better outcomes.[4] Let $G(\cdot)$ be a non-decreasing *probability weighting function* $G(P) : [0,1] \to [0,1]$ such that $G(0) = 0$ and $G(1) = 1$. The RDEU functional and the corresponding certainty equivalent are defined as:

$$V(c) = \int u(c)dG(P) \quad , \quad \mu = u^{-1}(V) \tag{3}$$

[4] In the atemporal case, for a monotone utility function, ranking of states is equivalently done by consumption or any monotone transformation of consumption. In a dynamic setting, x may be different from consumption as other state variables may contribute to preference ranking of states and consumption is only one of them.

It is convenient to illustrate the intuition for this utility in the discrete state case. Let $i = 1, ..., N$ index the outcomes ordered from the worst to the best. Then the utility function is given by:

$$V(c) = \sum_{i=1}^{N-1} u(c_i)(G(P_i) - G(P_{i+1}))$$

The utility is weighted by *decision weights* $q_i \equiv G(P_i) - G(P_{i+1})$ which add up to one (by design) and serve as a transformed probability measure. We emphasize however, that decision weights are not a model of biased or subjective beliefs. The beliefs of the decision maker are assumed to be unbiased and coincide with the objective probabilities. The purpose of the decision weights is to model separate risk attitude to probabilities of events as opposed to risk attitude towards level of consumption captured by $u(\cdot)$. The mechanism of decision weights is similar to the standard utility u mapping consumption into utility values but applied to probabilities. Note that if we denote $Z(P) = G'(P)$ we can rewrite the decision weight as $w_i \approx p_i Z(P_i)$ ($p_i = P_i - P_{i-1}$, $P_0 = 0$). Observe that the events with $Z > 1$ are overweighted and the events with $Z < 1$ are underweighted relative to their objective probability p_i. When $G(P) = P$, the model coincides with EU. Since G is a probability transformation and $G(1) = 1$ and $G(0) = 0$, we have $1 = \int dG(P) = \int Z(P)dP = E\{Z\}$. Thus, on average, probability weighting density is equal to one. However, some events may be more important for the agent and receive higher weight in the utility function at the expense of less important events. For example, in the empirical analysis we consider a function G which overweights probabilities of bad events.

The disappointment aversion certainty equivalent μ is defined implicitly as follows:

$$u(\mu) = \int u(x)dP - A \int_{x \leq \mu} (u(\mu) - u(x))dP = E\left\{ u(x) \frac{(1 + AI(x \leq \mu))\}}{1 + AE\{I(x \leq \mu)} \right\} \tag{4}$$

where x is a random variable with cdf P, $I(\cdot)$ is the indicator function which is equal to one if condition is true and is zero otherwise, and $A > 0$ is the disappointment aversion parameter. Note from the above formula that disappointing events are defined as those with $x < \mu$ and these events are weighted by $\frac{1+A}{1+AE\{x<\mu\}}$ compared to the weight of $\frac{1}{1+AE\{x<\mu\}}$ for outcomes above μ. When $A = 0$, the model coincides with EU.

To derive asset pricing implications for the above utility functions, we rely on the representative agent model from Epstein and Zin (1989) which encompasses a broad class of homogeneous certainty equivalents, including DA and RDEU. The reader is referred to their paper for assumptions and technical details. Consider an infinitely-lived representative agent with access to N securities with the returns vector $R_t = [R_{1,t}, ..., R_{N,t}]'$. Let R_t^M denote the return on the optimal (market) portfolio and define x_{t+1} as follows:

$$x_{t+1} = \beta^{\frac{1}{\rho}} \left(\frac{c_{t+1}}{c_t}\right)^{1-\frac{1}{\rho}} (R_{t+1}^M)^{\frac{1}{\rho}} \tag{5}$$

Epstein and Zin (1989) show that the optimality condition for recursive preferences in (1) with respect to consumption choice is given by:

$$\mu_t(x_{t+1}) = 1 \tag{6}$$

where μ_t is the certainty equivalent computed at time t. We specialize in the case of power utility $u(x) = \frac{x^\alpha}{\alpha}$ with $0 \neq \alpha < 1$ ($u(x) = log(x)$, $\alpha = 0$). For the DA case, as shown in Epstein and Zin (2001), portfolio optimization implies the following Euler equations for returns:

$$E_t\{(1 + AI(x_{t+1} \leq 1))x_{t+1}^\alpha (R_{t+1}^M)^{-1} R_{i,t}\} = E_t\{1 + AI(x_{t+1} \leq 1)\} \quad , \quad i = 1, ..., N \tag{7}$$

The equation (7) can be rewritten in the standard SDF form $E_t\{M_{t+1}R_{i,t}\} = 1$ by defining the SDF as follows:

$$M_{t+1} = x_{t+1}^\alpha (R_{t+1}^M)^{-1} \frac{1 + AI(x_{t+1} \leq 1)}{1 + AE_t\{I(x_{t+1} \leq 1)\}} \tag{8}$$

By taking unconditional expectation of (7) we can also use the Euler equation in unconditional form with the corresponding change in the denominator of M_{t+1}.

In Appendix A we show that the SDF for RDEU is given by:

$$M_{t+1} = x_{t+1}^\alpha (R_{t+1}^M)^{-1} Z_{t+1} \tag{9}$$

where $Z_{t+1} = Z(x_{t+1})$ was introduced earlier as the density of the probability weighting function G applied to the cdf of x_{t+1}. Note that the first two terms in expressions (8) and (9) correspond to the EU-based SDF (Epstein-Zin recursive utility). We can test the EU hypothesis via a parametric restriction on each model.

Before proceeding to estimation we select a probability weighting function G to characterize preferences for atemporal risk under RDEU. We consider a piece-wise linear function with two parameters. One parameter $0 < p_0 < 1$ characterizes the cutoff for bad events, i.e. events with cumulative probability below p_0 are overweighted by the agent. Another parameter $0 \leq \phi \leq 1$ is characterizing the magnitude of overweighting. The functional form of G is given by:

$$G(P) = \begin{cases} \left(1 + \frac{1-p_0}{p_0}\phi\right) P & , \quad P < p_0 \\ (1-\phi)\,P + \phi & , \quad P \geq p_0 \end{cases} \tag{10}$$

This specification has two main advantages.[5] First, two parameters allow separate control over what outcomes are considered bad and how strongly they are overweighted. Second, the overweighting is uniform which reduces the influence of outliers in the left tail of the distribution.[6] Note that, when $\phi = 0$ we have $G(P) = P$ which corresponds to EU.

3 Empirical method and data

To estimate the model with SDFs given by (8) and (9) we use the Generalized Method of Moments (GMM). We use total of 27 test assets: 25 Fama-French portfolios sorted by size and book-to-market ratios, the risk-free asset (short term Treasury bills), and the value weighted market portfolio. Consumption growth rates and returns are at monthly frequency. As a proxy for total wealth return in the SDF we follow the traditional approach and use the value weighted market index return. We define the moment error function as follows:

$$g(\theta) = E\{m_t(\theta)R_t - 1\} \tag{11}$$

where R_t is the vector of returns on test assets and $m_t(\theta)$ is the stochastic discount factor for DA and RDEU models given in (8) and (9). The parameter vectors are $\theta = [\alpha, \sigma, \beta, A]$ for DA and $\theta = [\alpha, \sigma, \beta, \phi, p_0]$ for RDEU. To compute the SDF for RDEU we use empirical cdf (step function) for the cumulative density of state variable x_t recomputed at each iteration. As the first-stage weighting matrix for GMM we use identity matrix. The GMM estimator is defined in a standard way as:

$$\hat{\theta} = \arg\min_{\theta} g'(\theta)Wg(\theta)$$

We use Newey-West method with 12 month lag to compute standard errors of the estimates. The fit of the models is evaluated using the χ^2 distance test. We also compare the downside risk models

[5]Non-differentiability of $G(P)$ does not present a problem as long as it occurs in a finite number of points and state variable has a continuous density.

[6]For example another possible specification $G(P) = P^{\phi}$, has infinite derivative at zero and the left tail outliers are strongly overweighted. This specification also combines in one parameter the strength of overweighting and cutoff for bad events.

with a linear factor CAPM using Fama-French 3-factor model with a constant, market index excess return, SMB, and HML as factors. This linear model is also estimated by GMM by specifying $m_t = b' f_t$ where b is the vector of four coefficients and f_t is the vector of factors.

Monthly non-durables consumption data (seasonally adjusted) is obtained from NIPA (Table 2.8.5) from the Bureau of Economic Analysis website (www.bea.gov). The inflation adjustment is using the corresponding NIPA price index (Table 2.8.4). The growth rates of consumption are constructed from 1959:02 to 2009:12. The monthly returns for 25 Fama-French portfolios, Fama-French factors, market index return and the t-bill rates are obtained from Kenneth French data library on his web page at Dartmouth College.[7] The returns are for portfolios of stocks sorted into quintiles by the value of market equity and by book-to-market ratios. The returns are adjusted for inflation. I follow standard convention and refer to quintile B1 as *growth* stocks, quintile B5 as *value* stocks, quintile S1 as *small* stocks, and quintile S5 as *large* stocks.

4 Estimation results

The standard measure of fit for the GMM is the χ^2 distance. However, one of the traditional ways to evaluate an asset pricing model is to use the R^2 in the cross-sectional regression of average realized returns versus predicted expected returns based on the covariances with SDF. While the present model is nonlinear, we can still compute the R^2. We use the moment condition to compute the predicted expected return based on the covariance with the estimated SDF:

$$E\hat{R}_i = \frac{1}{E\{m(\hat{\theta})\}} - \frac{1}{E\{m(\hat{\theta})\}} Cov(m(\hat{\theta}), R_i) \tag{12}$$

We can then use the estimated returns for test assets $i = 1, \ldots, 27$ (first two assets are risk-free rate and market index return) and compute the R^2 for the cross-sectional regression as follows:

$$R^2 = 1 - \frac{\sigma^2(E\hat{R}_i - ER_i)}{\sigma^2(ER_i)}$$

where $E\hat{R}_i$ and ER_i are, respectively, predicted and realized average returns on test asset i. We report these R^2's along with χ^2 distances and the estimated coefficients.

Table 1 shows the results for CCAPM with EU, CCAPM with downside risk (RDEU and DA), and for a linear three-factor CAPM. We see that CCAPM based on EU is largely inconsistent with

[7]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the cross section of expected returns. While this specification have been used to address the risk premium and the risk free rates puzzles, the cross section of returns presents an additional challenge. The results show that there is not enough variation in the covariances of the EU-based SDF with test assets. The R^2 of this model is about 22% and the overidentifying restrictions are strongly rejected. The estimated risk aversion $(1-\alpha)$ is quite high, about 71. The EIS is close to zero at 0.022, however a test of $\sigma = 0$ is rejected (unreported). The subjective discount rate $\beta = 0.98$ is below one but not statistically different from one.

The next two panels of table 1 show the estimates for the downside risk models. Both specifications show considerable improvement compared to the EU-CCAPM. RDEU specification has a reasonably high R^2 of about 64% and its overidentifying restrictions cannot be rejected (p-value of 0.43). DA specification has R^2 of 63% and the distnace test fails to reject pricing restrictions (p-value of 0.77). The restrictions to EU are strongly rejected for both models as shown by the χ^2 tests in the last column. Parameter estimates indicate that utility curvature required to fit the models is much lower compared to the EU case, u is close to log or moderately risk averse.[8] The reason is that these models accomodate risk aversion through the probability weighting channel and it turns out to be more efficient than the standard utility curvature. The estimated coefficients of EIS are low, similar to the EU case. The time preference parameters are lower than in the EU case. This is because, compared to the EU, the states with high value of the SDF are overweighted and lower time preference is needed to maintain the mean of the SDF close to the risk-free rate.

How strong is the overweighting of bad events? To compute the overweighting for DA we use point estimates for parameter A and time series for the associated indicator function. Using this information we find that low outcomes are overweighted by a factor of 4.91 $(= \frac{1+A}{1+E\{AI(x_t \leq 1)\}} \approx \frac{10.86}{2.21})$ relative to the objective probability while the high outcomes are underweighted by a factor of approximately 0.45 $(\approx \frac{1}{2.21})$. For RDEU these factors are comparable, overweighting by 4.79 $(\approx 1 + \frac{1-0.15}{0.15} \cdot 0.67 \approx 1 + \frac{1-p_0}{p_0}\phi)$ while underweighting is 0.33 $(= 1 - 0.67 = 1 - \phi)$. In the case of DA the overweighting applies to the events approximately in the 8-th percentile of the distribution of the state variable x_t and in the case of RDEU it applies to the events approximately in the 14-th percentile. In section 5 we will discuss the implications of these parameters.

[8]While the standard errors are high for this parameter, risk neutrality $(\alpha = 1)$ is rejected. A detailed examination of the GMM objective function shows that it is relatively flat with respect to the risk aversion near the point estimate, resulting in high standard errors.

Finally, Table 1 also shows the results for the Fama-French linear 3-factor model. On the R^2 metric the linear model appears to be better than both specifications of CCAPM with downside risk aversion. However, based on the χ^2 distance test, the linear model fit is worse and its overidentifying moment restrictions are rejected. Thus, while on average the linear model fits the cross-section of expected returns better, taking into account the variability of pricing errors puts it behind either RDEU- or DA-based CCAPM.

Table 2 shows the pricing errors. EU-CCAPM misses most of the expected returns and over predicts the returns on the risk free asset and the market index by 30 and 17 basis points (bp) respectively. RDEU and DA based CCAPM have considerably smaller pricing errors, although a bit higher than the FF 3-factor CAPM. For the market return, downside risk models prediction is closely matching that of the FF model. The risk free rate of return is notoriously difficult to match but RDEU model comes within 5 bp while FF model is within 10 bp. Both DA- and EU-CCAPM have substantial pricing error for the risk free asset.[9] Note also that downside risk models have lower pricing errors for small growth portfolio (S1B1) compared to EU-CCAPM or 3-factor CAPM.

Our results so far indicate that integrating aversion to downside risk considerably improves CCAPM fit, and that the downside risk models are either comparable to (on the R^2 basis) or better than (on the χ^2 distance basis) the Fama-French linear 3-factor model. The results are also noteworthy due to several additional considerations. First, the models are estimated at monthly frequency. Consumption-based models tend to provide a better cross-sectional fit when estimated at longer horizons of several quarters (Parker and Julliard (2005), Jagannathan and Wang (2007)). At quarterly frequency one would typically need additional conditional variables to generate performance at par with a linear factor model with returns-based factors (Lettau and Ludvigson (2001)). Second, the sample includes the dot-com bubble collapse and 2008-2009 financial crisis, which are difficult to explain in a standard consumption-based framework. Third, downside risk models, while having similar number of parameters as the linear factor model, only use consumption and market return compared to the three return-based factors in the linear model. Finally, we explicitly include the risk free return and the downside risk models can generate reasonable implications for it. Despite these additional hurdles, the results present strong support for the importance of downside risk in

[9] In the base case in table 1, DA aversion has a relatively high pricing error for the t-bill rate. We found that this error varies across specifications and in some other cases it was small, comparable to RDEU. However, in our tests, the EU-CCAPM always considerably overpredicted the risk free rate.

the cross-section of returns. In the rest of the paper, we investigate the economic intuition behind the models with downside risk aversion and also demonstrate robustness of our main results.

5 Exploring the results

In this section, we explore the economic intuition behind the results for CCAPM with aversion to downside risk. We first evaluate how reasonable are the implications of the parameters obtained in estimation. To do that, we show the implications of RDEU and DA models for a static portfolio choice problem with two assets and compare them to the implications of EU. To select suitable parameter ranges we use point estimates and standard errors from the baseline case and industry and subset of portfolios estimation presented later in the robustness section. The portfolio choice problem is very simple, one asset is a risk free asset with a constant return and another asset is a risky stock with lognormally distributed return.[10] We assume that investor has a power felicity function $u(w) = w^{\alpha}/\alpha$ and solve the problem numerically for various parameters. The results are reported in table 3.[11] We observe that portfolio shares implied by RDEU and DA are generally conservative and are comparable to EU with risk aversion of about 5 to 10. For higher values of ϕ for RDEU and A for DA the optimal policy is to hold no stocks. This, of course, does not suggest that a representative agent would not hold stocks! This implications is only limited to a static problem without intermediate consumption. In an infinite horizon problem, the agent's total wealth is given by the capitalized value of consumption stream. Since consumption is not very risky, holding stocks is attractive and the estimated Euler equations imply, by construction, that stocks are optimally held. Also noteworthy is that both A and ϕ have high standard errors across all specifications and while we can reject that they are equal to zero (EU case), distinguishing between moderate and higher values is difficult. Thus, we could only say that the range of parameters obtained in estimation appears to be reasonable and implies moderate to conservative portfolio allocations.

We now proceed to analyze the structure the SDFs for the downside risk models. We can write both SDFs as the product of EU-based SDF m_t^0 and an additional factor corresponding to the

[10] We set the risk free rate to 3% and the mean of the log stock return to 10% and the standard deviation to 18%.

[11] The optimality condition for RDEU is given by $E\{u'(R_f+\alpha^*(R_S-Rf))Z(R_S)(R_S-Rf)\}=0$. We use numerical integration and standard procedure (*fzero*) in MATLAB to find the solution. For optimal condition for DA and its solution method, see Ang, Bekaert and Liu (2005).

downside risk:

$$\begin{aligned} m_t &= m_t^0 y_t \\ m_t^0 &= x_t^\alpha (R_t^M)^{-1} \\ y_t &= Z(x_t) \quad \text{, for RDEU} \\ y_t &= \frac{1 + AI(x_t \le 1)}{1 + AE\{I(x_{t+1} \le 1)} \quad \text{, for DA} \\ x_t &= \beta^{\frac{1}{\rho}} \left(\frac{c_t}{c_{t-1}} \right)^{1-\frac{1}{\rho}} (R_t^M)^{\frac{1}{\rho}} \end{aligned}$$

Note that $E\{y_t\} = 1$ for both models and parametric restriction to EU result in $y_t \equiv 1$. In the unrestricted cases y_t amplifies the effect of bad economic states. What constitutes a bad sate? Consider the state variable x_t and the optimality condition in (6). As Routledge and Zin (2010) note, in a deterministic economy the optimality implies that $x_t = 1$. They refer to x_t as *ex post* savings error. Consider the optimal consumption growth in the deterministic economy:

$$1 = \beta R_t^M \left(\frac{c_{t+1}}{c_t} \right)^{\rho - 1} \Rightarrow \frac{c_{t+1}}{c_t} = \left(\beta R_t^M \right)^\sigma$$

where we substitute $\frac{1}{1-\rho} = \sigma$. In the stochastic case for DA, the overweighted bad sates are those with $x_t < 1$. If $\sigma < 1$ then $\rho = 1 - \frac{1}{\sigma} < 0$ and we have in bad states:

$$1 > \left(\beta R_t^M \left(\frac{c_{t+1}}{c_t} \right)^{\rho - 1} \right)^{\frac{1}{\rho}} \Rightarrow \frac{c_{t+1}}{c_t} < \left(\beta R_t^M \right)^\sigma \tag{13}$$

Therefore bad states are those with low consumption growth rate *relative* to the adjusted total wealth return. Such states occur either when consumption growth is low or when the adjusted total wealth return is higher than consumption growth. It is intuitive that states with low consumption growth are overweighted. To understand the intuition for the other case, recall the optimal condition for consumption given in (6). It requires that the certainty equivalent of the state variable x_t be equal to one. Therefore, for states with consumption growth below the threshold in (13) there must be states when the growth is higher than the threshold. Because the agent values smooth consumption paths, the deviations from the optimal growth are undesirable, even when they are a result of unexpectedly high portfolio returns. The intuition is the same for RDEU, but the bad events cutoff is a parameter, so the overweighted events are those with $x_t < P^{-1}(p_0)$ where $P^{-1}(\cdot)$ is the inverse cdf of x_t.

There is some similarity of the downside risk models SDFs with the habit formation model of Campbell and Cochrane (1999). For the habit model, the SDF is a function of consumption growth and external habit process h_t:

$$m_{t+1} = \left(\frac{c_{t+1}}{c_t}\right)^{-(1-\alpha)} \left(\frac{s_{t+1}}{s_t}\right)^{-(1-\alpha)}$$

where the surplus ratio is defined as $s_t = \frac{c_t - h_t}{c_t}$. The surplus ratio falls during recessions because habit stock responds with lag to consumption. As a result, the SDF overweights the marginal utility when consumption growth is low by an additional factor and underweights it in normal or high growth states. The downside risk models have a similar mechanism, but with a different state variable. Habit formation model has been used to motivate additional risk factors in standard CCAPM (Lettau and Ludvigson 2001, Parker Julliard 2005). However, unlike the models considered in this paper, habit formation utility is not an axiomatic model of preferences and is not derived from the observed investor behavior. While it is a convenient analytical tool, it does not explain what exactly generates the two-component SDF and what features of investor behavior are responsible for this structure. The advantage of the models considered in this paper is that they provide intuitive interpretation of asset prices which can be traced to behavioral assumptions used to derive risk preferences.

One important difference between the habit model and the recursive utility considered here is that the latter, by itself, does not generate conditional heteroskedasticity of the SDF necessary for time-varying risk premium. However, some alternative mechanisms can generate this property. A detailed investigation of this interesting issue is outside the scope of this paper, but we offer some related observations. Bansal and Yaron (2004) consider EU-based recursive utility and directly assume that aggregate consumption growth is heteroscedastic. This generates sufficient time variation in risk premium in their model. The models with DA- and RDEU-based recursive utility are also amenable to this approach. One may reasonably conjecture that over-weighting of bad states further amplifies the effect of heteroscedasticity in consumption on asset prices.[12] Direct assumption of heteroskedasticity in consumption growth rates may be also replaced by an endogenous mechanism of learning about a hidden state. One such example is provided in Ju and Miao (2010) where

[12]This is because in lower growth states the distribution of future consumption growth shifts to the left due to the persistent growth shock in the long-run risk model. The left tail is overweighted and therefore bad outcomes have even more significance in the SDF, raising its volatility in the downturns. The weighting function is flatter for high outcomes and this may dampen the volatility when higher future growth is expected.

Bansal and Yaron's model is extended to include learning the unobservable state of the aggregate endowment growth rate. The SDF considered by Ju and Miao has a similar structure to RDEU or DA-based SDFs and overweights "bad states" in the distribution of a state variable. While Ju and Miao assume that endowment is homoskedastic, due to learning, their model produces reasonable time variation in expected returns. A similar mechanism can be employed in the models considered here.

We now explore the sources of variation in cross-sectional risk premium generated by the downside risk models. Consider again the pricing equation implied by the downside risk models. Denote $\mu_0 = Em_t^0$ and recall that $Ey_t = 1$. We can write a decomposition of the expected risk premium on portfolio i $ER_{t,i}^e$ $(= ER_{t,i} - r_f)$ as follows:

$$
\begin{aligned}
0 &= E\left\{m_t^0 y_t R_{t,i}^e\right\} \\
0 &= E\left\{((m_t^0 - \mu_0) + \mu_0)((y_t - 1) + 1)R_{t,i}^e\right\} \\
0 &= \mu_0 ER_{t,i}^e + E\left\{(m_t^0 - \mu_0)R_{t,i}^e\right\} + \mu_0 E\left\{(y_t - 1)R_{t,i}^e\right\} + E\left\{(m_t^0 - \mu_0)(y_t - 1)R_{t,i}^e\right\}
\end{aligned}
$$

The last equation implies the following decomposition of risk premium:

$$
ER_{t,i}^e = \underbrace{-(1/\mu_0)E\left\{(m_t^0 - \mu_0)R_{t,i}^e\right\}}_{\text{Consumption risk (EU-based)}} \underbrace{-E\left\{(y_t - 1)R_{t,i}^e\right\}}_{\text{Downside risk}} \underbrace{-(1/\mu_0)E\left\{(m_t^0 - \mu_0)(y_t - 1)R_{t,i}^e\right\}}_{\text{Co-movement of EU-based SDF and downside risk}}
$$

The first term is the familiar covariance of excess return with EU-based SDF m_t^0. For the downside risk models there are two additional terms. The first one is the covariance of excess return with y_t. Since y_t increases in states which consumer perceives as bad, the assets that tend to have low excess returns in those states will have a negative covariance with y_t and a positive downside risk premium. The third term captures co-movements of excess return with the product of the EU-based SDF and the downside risk component. While both m_t^0 and y_t depend on the same variables, they contain different type of information about the pricing kernel and represent different sources of risk. y_t contains information about the state's relative importance among all possible states, while the EU portion contains information about marginal utility in a given state.

Using the estimates of the models, we can compute consumption and downside risk premium components for the 25 Fama-French portfolios. Table 4 shows the components of the risk premium for both models. Growth stocks tend to have lower downside risk component than value stocks. The change in the average risk premium across portfolios going from the growth quintile (1) to the value quintile (5) is about 20 bp (RDEU) or 30 bp (DA). On the size dimension, the average downside risk premium change from large to small portfolios by 12 bp (RDEU) to about 17 bp (DA). On the other hand, consumption risk premium does not exhibit much variation across portfolios.

The models with aversion to downside risk emphasize the distinction between value and growth stocks and small and large stocks in their correlation with downside risk component of the SDF. Value stocks and small stocks do relatively poorly in recessions and must have a higher risk premium. This intuition is consistent with empirical research on conditional CCAPM (e.g. Lettau and Ludvigson (2000)). Unlike the conditional CCAPM however, the models here explicitly link downside risk to variation in risk premium. In summary, a relatively successful performance of the downside risk models has intuitive economic interpretation. Their SDFs emphasize the risk of states which are undesirable to economic agents. The variation in risk premia on test portfolios is explained in part by the portfolios' covariances with the downside risk component of the SDF.

6 Some robustness analysis

A critical issue arising in linear factor models estimated using size and book-to-market portfolios is that these portfolios have a strong two-factor structure as discussed by Lewellen, Nagel and Shanken (2010). As a result, some linear factor models may appear to provide a good cross-sectional fit based on the R^2 even if the explanatory power of the risk factors is low or spurious. Lewellen *et al.* (2010) develop simulation-based methods for evaluating significance of the cross-sectional R^2's in the linear factors models and also recommend including additional portfolios to test robustness of the factors.

The problem of spurious factors is less of an issue in the present context with a single nonlinear stochastic discount factor. Lewellen *et al.* (2010) show that for a one-factor model it would be very unlikely to obtain spuriously good cross-sectional fit because a single factor would have to have joint information on size and book-to-market risk premia. For example, based on the simulations presented in their paper on figure 2 (and figure 5), a single-factor model which has in sample R^2 of 0.6 implies a 95% confidence interval for the true cross-sectional R^2 between approximately 0.5 and 1. Also note that significance of the downside risk parameters is measured by the χ^2 statistic which is more informative than R^2.

Nevertheless, to assuage any potential concerns, we re-estimate the models on industry portfolios.

In these tests we use data for 17 industry portfolios obtained from Kenneth French web data library. The results reported in table 5 are consistent with previous findings. Downside risk is highly significant for both RDEU and DA models and their restrictions to EU are strongly rejected. Downside risk models continues to provide considerably better fit than either EU-based model or 3-factor CAPM based on distance tests. Their overidentifying restrictions are not rejected. The cross-sectional R^2s deteriorate for all models, including the Fama-French 3-factor model. This is consistent with Lewellen *et al.* (2010) argument that R^2 may be a misleading metric and a more stringent test must include portfolios other than book-to-market and size sorted portfolios.

It is interesting to see how the SDFs with downside risk compare with 3-factor CAPM when both downside risk SDF and the return-based factors are allowed to explain the cross section of returns. Since the models are non-nested we first construct factor mimicking portfolios using the point estimates of parameters from table 1. Factor mimicking portfolios application to consumption-based model tests was introduced by Breeden, Gibbons and Litzenberger (1989). We construct portfolio weights by projecting a nonlinear SDF on the matrix of test asset returns. The factor mimicking portfolio return is then combined with the Fama-French factors in a linear factor model. Table 6 (Panel A) shows the results for in-sample estimation. We observe that downside risk factors are highly significant when inserted alongside the Fama-French factors. Moreover, the presence of a downside risk factor shrinks the values of the Fama-French portfolio coefficients towards zero and reduces their statistical significance. A Wald test for jointly zero coefficients on SMB and HML factors is near 5% margin for RDEU and is not rejected at 5% confidence for DA.

Factor mimicking portfolios are also useful for extending the analysis of the models out-of-sample. We use the projection weights obtained in-sample to construct factor mimicking portfolios for the full sample of CRSP monthly returns from 1929:07 to 2009:12. We then repeat the previous exercise. The results are presented in Table 6 (Panel B) along with the re-estimated Fama-French model coefficients for the full sample. We observe that in the full sample, the downside risk factors are the only statistically significant factors. Wald tests for SMB and HML coefficients jointly equal to zero are not rejected at 10% confidence for either model. In summary, our analysis based on factor mimicking portfolios confirms that downside risk is an important component of risk premium and that it is related to size and value premiums in the Fama-French portfolios. The out-of-sample extension of the original downside risk factor portfolios confirms their properties are robust.

To further evaluate the stability of the model with respect to the choice of test assets, I use a subset of 9 portfolios from the original 25. I select 9 portfolios that represent value and size premium: B1S(1,3,5), B3S(1,3,5), B5S(1,3,5). Table 7 reports the results for these portfolios We observe that with fewer portfolios the cross-sectional R^2's are somewhat better than in the baseline specification, and the χ^2 distances are lower which indicates lower pricing errors for this subset of portfolios. The coefficient estimates are consistent with those in Table 1. The tests of EU-based restrictions are, again, strongly rejected. In addition to the reported tests, other robustness tests included: 10 industry portfolios, combination of industry portfolios (both 10 and 17) with a subset of 10 Fama-French portfolios (B1S1-5;B5S1-5), other subsample tests. The results are qualitatively similar and are omitted for brevity.

7 Conclusions

We consider two consumption-based models which incorporate aversion to downside risk and find that this feature is empirically relevant for characterizing risk premia in financial assets. Our results show intuitive decomposition of risk premium and we find that the portion attributed to the downside risk contributes to the size and value premium. An implication of our results is that standard mechanism based on marginal utility of consumption cannot fully account for risk premium and a downside risk factor is required. This implication provides a theoretical foundation for and explains successful performance of a large class of empirical models which use additional factors to proxy for downside risk. Our results also provide direct non-experimental support for investor behavior captured by the utility functions with downside risk aversion. This adds considerable credibility to many other interesting empirical predictions of such models and hopefully will encourage more applications in future research.

Appendix A Euler equation for RDEU

In this appendix, we derive Euler equation for recursive utility when certainty equivalent is given by RDEU. Consider rank-dependent utility function given by:

$$v(y) = \int \frac{y^\alpha}{\alpha} Z(P) dP \tag{A.1}$$

where P is the cdf of y and $Z = G'(P)$ is the probability weighting density. As shown in Epstein and Zin (1989, p. 957) the optimal portfolio weights w for the recursive utility solve the following maximization problem:

$$\max_w v(x_{t+1}(R^M_{t+1})^{-1}w'R_{t+1}) \tag{A.2}$$

where x_t is the state variable given in (5) and R^M_t is the return on the market portfolio (total wealth portfolio). To differentiate (A.2) with respect to w we use the results from Ai (2005) and Carlier and Dana (2003) who prove differentiability of RDEU functional with respect to continuously distributed random variables. Consider a random variable s added in the amount a to y. Then:

$$\frac{\partial v}{\partial a}(y + as) \mid_{a=0}= \int v'(y)sZ(P)dP \tag{A.3}$$

Taking into account portfolio constraint $w'\iota = 1$ this implies that the first order condition for optimal portfolio weights in (A.2) is given by:

$$E_t\{(x_{t+1})^\alpha Z(P)(R^M_{t+1})^{-1}(R_{j,t+1} - R_{i,t+1})\} = 0 \quad , \quad j \neq i \tag{A.4}$$

To obtain Euler equation for return rewrite (A.4), multiply by weights, and sum up to obtain:

$$E_t\{(x_{t+1})^\alpha Z(P)(R^M_{t+1})^{-1}R_{i,t+1}\} = E_t\{(x_{t+1})^\alpha Z(P)\} = 1 \quad , \quad i = 1, ..., N \tag{A.5}$$

where the last equality follows from consumption optimality equation $\mu_t(x_{t+1}) = 1$ (see Epstein and Zin (1989) p 958 eq. (6.3)). Thus, we can write the optimal condition in the SDF form as follows:

$$\begin{aligned} m_{t+1} &= (x_{t+1})^\alpha Z(P)R^M_{t+1})^{-1} \\ 1 &= E_{t+1}\{m_{t+1}R_{i,t+1}\} \quad , \quad i = 1, ..., N \end{aligned}$$

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Table 1: The table reports estimates for CCAPM with EU, RDEU and DA specifications and for Fama-French 3-factor CAPM. The models are estimated using monthly data from 1959:02 to 2009:12. Test assets include t-bill rate, value-weighted CRSP market index return, and 25 Fama-French portfolios. Consumption growth is computed from monthly series of nondurables obtained from NIPA. Standard errors are given in parenthesis. The table also reports cross-sectional R^2 for each model and the χ^2 test of overidentifying restrictions with the corresponding p-value which is reported in brackets below the statistic. In the last column for RDEU and DA models, the table shows a χ^2 test of parameter restriction corresponding to EU, the corresponding p-value is shown in brackets below the statistic.

			CCAPM (EU)				
α	σ	β			R^2	χ^2	
-70.45	0.022	0.979			0.225	107.7	
(33.9)	(0.019)	(0.052)				[0.000]	
			CCAPM (RDEU)				
α	σ	β	ϕ	p_0	R^2	χ^2	$\chi^2(\phi = 0)$
-0.836	0.021	0.81	0.665	0.149	0.640	22.4	85.2
(91.5)	(0.001)	(0.020)	(0.452)	(0.016)		[0.434]	[0.000]
			CCAPM (DA)				
α	σ	β	A		R^2	χ^2	$\chi^2(A = 0)$
-0.395	0.034	0.825	10.5		0.630	17.7	89.9
(46.2)	(0.001)	(0.017)	(14.7)			[0.773]	[0.000]
			CAPM (FF 3-factor)				
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)		R^2	χ^2	
1.046	-2.63	-2.53	-7.578		0.752	89.9	
(0.023)	(1.263)	(1.695)	(2.145)			[0.000]	

Table 2: Pricing errors for 25 Fama-French portfolios, risk-free asset return and market index return. Pricing errors are in percentage points computed as the average realized return minus the return predicted by the model using the estimates from Table 1. Averages across portfolios are computed using the absolute value of pricing errors for each portfolio in a given quintile.

	EU-CCAPM						RDEU-CCAPM					
	Small	2	3	4	Big	Abs. Avr.	Small	2	3	4	Big	Abs. Avr.
Growth	0.32	0.27	0.28	0.19	0.19	0.25	0.08	0.29	0.21	0.13	0.11	0.16
2	-0.07	0.07	0.07	0.10	0.17	0.10	0.05	-0.03	0.06	0.10	0.06	0.06
3	-0.08	-0.24	-0.03	0.06	0.07	0.10	0.02	-0.25	-0.06	0.12	-0.09	0.11
4	-0.40	-0.27	-0.15	-0.07	0.24	0.23	-0.27	-0.25	0.08	0.12	0.12	0.17
Value	-0.48	-0.27	-0.29	-0.00	-0.14	0.24	-0.29	-0.18	-0.19	0.04	-0.08	0.16
Abs. Avr.	0.27	0.22	0.16	0.09	0.16		0.14	0.20	0.12	0.10	0.09	
	R_f	0.30	R_M	0.17			R_f	0.05	R_M	0.09		
	DA-CCAPM						FF 3-factor CAPM					
	Small	2	3	4	Big	Abs. Avr.	Small	2	3	4	Big	Abs. Avr.
Growth	0.16	0.19	0.10	0.05	-0.02	0.10	0.40	0.11	-0.01	-0.18	-0.20	0.18
2	0.06	0.01	0.01	0.12	0.03	0.05	-0.05	0.02	-0.08	0.07	-0.02	0.05
3	0.02	-0.21	-0.05	0.06	-0.07	0.08	-0.00	-0.11	0.00	0.01	0.04	0.03
4	-0.20	-0.25	0.03	0.06	0.12	0.13	-0.17	-0.08	-0.05	-0.05	0.18	0.11
Value	-0.27	-0.19	-0.19	-0.01	-0.07	0.15	-0.13	0.01	-0.05	0.11	0.18	0.10
Abs. Avr.	0.14	0.17	0.08	0.06	0.06		0.15	0.06	0.04	0.08	0.12	
	R_f	0.46	R_M	0.05			R_f	0.10	R_M	-0.02		

Table 3: The table shows optimal portfolio share invested in stock in a static portfolio chocie problem with two assets, a risk free asset with a constant return and a risky stock with lognormally distributed return. Calibration parameters are dsicussed in the text.

EU		RDEU ($\alpha = -1$)				DA	
α		p_0/ϕ	0.3	0.4	0.6	A	$\alpha = -1$
-1	1.7	0.15	0.56	0.24	0.00	1	0.50
-4	0.70	0.25	0.73	0.42	0.00	2	0.08
-9	0.35	0.35	0.93	0.65	0.04	5	0.00

Table 4: Consumption risk premium and downside risk premium.

	Consumption risk premium (RDEU)						Downside risk premium (RDEU)					
	Small	2	3	4	Big	Average	Small	2	3	4	Big	Average
Growth	0.29	0.29	0.27	0.25	0.20	0.26	-0.07	0.31	0.28	0.31	0.22	0.21
2	0.25	0.24	0.23	0.22	0.19	0.23	0.48	0.31	0.45	0.32	0.24	0.36
3	0.22	0.22	0.20	0.21	0.18	0.21	0.47	0.31	0.37	0.51	0.12	0.36
4	0.21	0.21	0.20	0.20	0.17	0.19	0.43	0.36	0.63	0.63	0.35	0.48
Value	0.22	0.23	0.21	0.21	0.18	0.21	0.49	0.50	0.50	0.53	0.25	0.45
Average	0.24	0.24	0.22	0.22	0.18		0.36	0.36	0.45	0.46	0.24	

	Consumption risk premium (DA)						Downside risk premium (DA)					
	Small	2	3	4	Big	Average	Small	2	3	4	Big	Average
Growth	0.29	0.29	0.27	0.25	0.20	0.26	-0.41	-0.20	-0.24	-0.18	-0.31	-0.27
2	0.25	0.24	0.23	0.22	0.19	0.22	0.08	-0.06	-0.00	-0.07	-0.20	-0.05
3	0.22	0.21	0.20	0.21	0.18	0.20	0.06	-0.05	-0.03	0.04	-0.27	-0.05
4	0.21	0.21	0.19	0.19	0.17	0.19	0.08	-0.04	0.17	0.16	-0.05	0.06
Value	0.22	0.23	0.21	0.21	0.18	0.21	0.11	0.08	0.09	0.07	-0.15	0.04
Average	0.24	0.23	0.22	0.22	0.18	0.00	-0.02	-0.06	-0.00	0.00	-0.19	0.00

Table 5: Estimates for industry portfolios. The table reports estimates for CCAPM with EU, RDEU and DA specifications and for Fama-French 3-factor CAPM. The models are estimated using monthly data from 1959:02 to 2009:12. Test assets include t-bill rate, value-weighted CRSP market index return and 17 industry portfolios. Consumption growth is computed from monthly series of nondurables obtained from NIPA. Standard errors are given in parenthesis. The table also reports cross-sectional R^2 for each model and the χ^2 test of overidentifying restrictions with the corresponding p-value which is reported in brackets below the statistic. In the last column for RDEU and DA models, the table shows a χ^2 test of parameter restriction corresponding to EU, the corresponding p-value is shown in brackets below the statistic.

			CCAPM (EU)				
α	σ	β			R^2	χ^2	
-13.2	0.012	1.136			0.202	31.3	
(30.5)	(0.083)	(0.92)				[0.012]	
			CCAPM (RDEU)				
α	σ	β	ϕ	p_0	R^2	χ^2	$\chi^2(\phi = 0)$
-0.110	0.011	0.812	0.343	0.346	0.332	18.0	13.2
(45.5)	(0.001)	(0.925)	(0.462)	(0.034)		[0.205]	[0.000]
			CCAPM (DA)				
α	σ	β	A		R^2	χ^2	$\chi^2(A = 0)$
-0.225	0.020	0.800	3.01		0.421	19.5	11.8
(12.2)	(0.001)	(0.014)	(2.37)			[0.191]	[0.001]
			CAPM (FF 3-factor)				
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)		R^2	χ^2	
0.997	-2.497	3.893	0.578		0.342	28.2	
(0.017)	(1.254)	(2.482)	(2.943)			[0.021]	

Table 6: Factor mimicking portfolios tests. The table reports estimates of the models using factor-mimicking portfolios. Factor-mimicking for RDEU and DA-based SDFs are constructed by projecting the SDF on the matrix of the original test assets returns. Factor mimicking portfolio return is then added to the Fama-French 3-factor model. Standard errors of the coefficients are shown in parenthesis. Wald tests statistics for the hypothesis $b_2 = b_3 = 0$ and associated p-values are reported in the last column.

Panel A: In-sample (1959:12-2009:12)					
CAPM (FF 3-factor)					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)		
1.046	-2.63	-2.53	-7.578		
(0.023)	(1.263)	(1.695)	(2.145)		
With RDEU factor					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)	b_4 (RDEU)	Wald ($b_2 = b_3 = 0$)
0.391	-0.659	-2.240	-4.936	0.637	6.42
(0.174)	(1.296)	(1.693)	(1.980)	(0.175)	[0.040]
With DA factor					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)	b_4 (DA)	Wald ($b_2 = b_3 = 0$)
0.502	-2.254	-1.127	-4.767	0.732	5.57
(0.207)	(1.217)	(1.757)	(2.016)	(0.209)	[0.062]
Panel B: Full-sample (1929:07-2009:12)					
CAPM (FF 3-factor)					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)		
1.006	-0.166	-1.856	-1.732		
(0.009)	(0.862)	(1.538)	(1.520)		
With RDEU factor					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)	b_4 (RDEU)	Wald ($b_2 = b_3 = 0$)
0.877	-0.887	-0.346	-2.360	0.137	4.310
(0.055)	(0.739)	(1.150)	(1.275)	(0.058)	[0.116]
With DA factor					
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)	b_4 (DA)	Wald ($b_2 = b_3 = 0$)
0.836	-1.288	0.066	-2.440	0.181	4.09
(0.064)	(0.733)	(1.124)	(1.241)	(0.068)	[0.130]

Table 7: Estimates for subset of portfolios. The table reports estimates for CCAPM with EU, RDEU and DA specifications and for Fama-French 3-factor CAPM. The models are estimated using monthly data from 1959:02 to 2009:12. Test assets include t-bill rate, value-weighted CRSP market index return, and 9 Fama-French portfolios: B1S(1,3,5), B3S(1,3,5), B5S(1, 3, 5). Consumption growth is computed from monthly series of nondurables obtained from NIPA. Standard errors are given in parenthesis. The table also reports cross-sectional R^2 for each model and the χ^2 test of overidentifying restrictions with the corresponding p-value which is reported in brackets below the statistic. In the last column for RDEU and DA models, the table shows a χ^2 test of parameter restriction corresponding to EU, the corresponding p-value is shown in brackets below the statistic.

CCAPM (EU)							
α	σ	β			R^2	χ^2	
-82.87	0.018	0.948			0.349	21.2	
(36.6)	(0.017)	(0.068)				[0.000]	
CCAPM (RDEU)							
α	σ	β	ϕ	p_0	R^2	χ^2	$\chi^2(\phi = 0)$
-0.726	0.021	0.864	0.783	0.148	0.793	4.30	16.9
(118.16)	(0.002)	(0.068)	(0.795)	(0.021)		[0.636]	[0.000]
CCAPM (DA)							
α	σ	β	A		R^2	χ^2	$\chi^2(A = 0)$
-0.347	0.031	0.809	9.19		0.702	11.7	9.49
(30.77)	(0.003)	(0.016)	(13.85)			[0.112]	[0.002]
CAPM (FF 3-factor)							
b_0	b_1 (R^M)	b_2 (SMB)	b_3 (HML)		R^2	χ^2	
1.049	-3.28	-1.444	-8.041		0.775	27.7	
(0.023)	(1.300)	(1.912)	(2.115)			[0.000]	