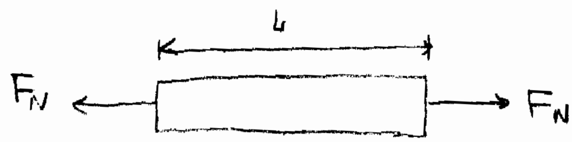
Strain Gauges MEEG3202 Spring 2006

1. Stress and stain

A. Uniaxial tension



$$\sigma_a = F_N / A$$

$$\varepsilon_a = \Delta L / L$$

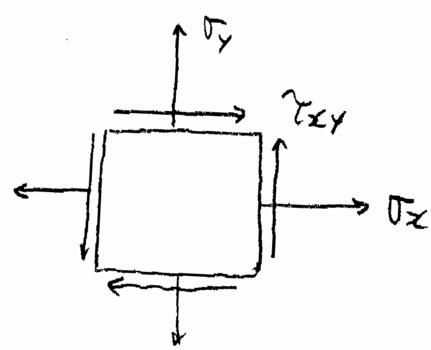
If the material deforms elastically, one has

$$\sigma_a = E\,\varepsilon_a \qquad E: \text{Young's Modulus}$$

Lateral strain (transverse strain) due to poisson ratio

$$\varepsilon_t = -\nu\,\varepsilon_a \qquad \nu = \frac{|\varepsilon_t|}{|\varepsilon_a|} \; : \text{poisson ratio}$$

B. Biaxial stress and strain



$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} \\ \varepsilon_y = \dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_x}{E} \\ \gamma_{xy} = \tau_{xy}/G \end{cases}$$

or

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\tau_{xy} = G\gamma_{xy}$$

$$G = \frac{E}{2(1+\nu)} \; : \text{shear modulus}$$

2. Strain Gauges

(i) Working principle

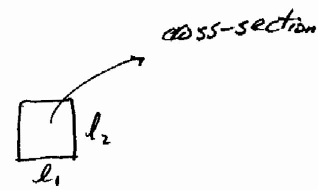
The electrical resistance (R) of a strain gauge changes upon deformation

Example: a conductor having a uniform area A, length l, made of a material having electrical resistivity ρ_e. The electrical resistance R is

$$R = \rho_e l / A$$

Assume that the gauge is subjected to a normal stress along the length direction, both the area and the length will change, so as the electrical resistivity. The electrical resistance is then

$$dR = \frac{l}{A} d\rho_e + \frac{\rho_e}{A} dl - \frac{\rho_e l}{A^2} dA$$



Note $A = l_1 \times l_2$

$$\frac{dA}{A} = \frac{dl_1}{l_1} + \frac{dl_2}{l_2}$$

The poisson effect gives

$$\frac{dl_1}{l_1} = \frac{dl_2}{l_2} = -\nu \frac{dl}{l} = -\nu \varepsilon_a$$

$$\therefore \quad dR = \frac{\rho_e l}{A} \frac{d\rho_e}{\rho_e} + \frac{\rho_e l}{A} \frac{dl}{l} - \frac{\rho_e l}{A} (-2\nu) \frac{dl}{l}$$

i.e. $$\frac{dR}{R} = \frac{d\rho_e}{\rho_e} + \frac{dl}{l}(1+2\nu)$$

observations: The change in resistance is caused by two basic effects

(1) change of geometry due to applied strain

(2) change of the resistivity ρ_e — piezo resistance

Define π as the piezo resistance coefficient

$$\pi = \frac{1}{E}\,\frac{d\rho_e/\rho_e}{dl/l} \Rightarrow \frac{d\rho_e}{\rho_e} = \pi E \cdot \frac{dl}{l}$$

$$\therefore \quad \frac{dR}{R} = \frac{dl}{l}(1+2\nu+\pi E)$$

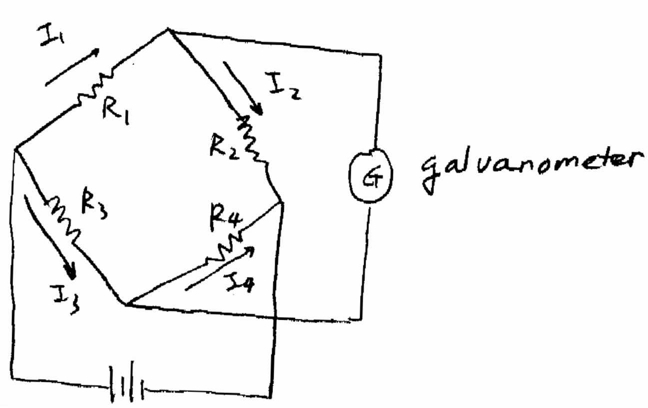
$$GF \equiv 1+2\nu+\pi E \quad : \text{Gauge factor}$$
$$= \frac{dR/R}{dl/l}$$

3. Strain Gauge electrical circuit (wheatstone bridge)

— detect small changes in electrical resistance due to strain

Two methods

(i) Null method



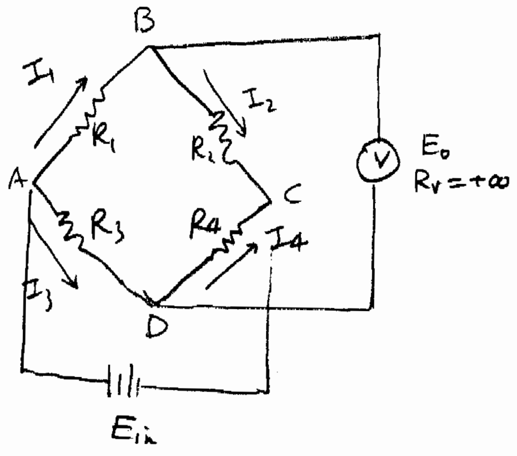
A. zero the galvanometer (no voltage drop)

$$\left.\begin{array}{l} I_1 R_1 - I_3 R_3 = 0 \\ I_2 R_2 - I_4 R_4 = 0 \\ I_1 = I_2, \quad I_3 = I_4 \end{array}\right\} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

B. Let R_1 be active (strain gauge), R_2 be adjustable, R_3 and R_4 are fixed. due to the applied strain. R_1 changes to $R_1 + \Delta R_1$, rezero the galvanometer by adjust R_2 to R_2', then

$$\frac{R_1 + \Delta R_1}{R_2'} = \frac{R_3}{R_4} \Rightarrow \Delta R_1 = \frac{R_3}{R_4} \cdot \Delta R_2$$

(ii) Deflection method



$$E_o = I_1 R_1 - I_3 R_3$$

$$I_1 = E_i / (R_1 + R_2)$$

$$I_2 = E_i / (R_3 + R_4)$$

$$\therefore E_o = E_i \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right)$$

A. Balance the bridge ($E_o = 0$)

$$\Rightarrow \frac{R_1}{R_1 + R_2} = \frac{R_3}{R_3 + R_4}$$

B. For a quater bridge, assume only R_1 is active, the change of E_o due to the applied strain is then

$$\delta E_0 = E_i \left[\frac{\delta R_1}{R_1+R_2} - \frac{R_1\, \delta R_1}{(R_1+R_2)^2} \right]$$

if $R_1 = R_2 = R$, then

$$\delta E_0 = E_i \left(\frac{\delta R}{2R} - \frac{\delta R}{4R} \right) = E_i \frac{\delta R}{4R}$$

$$\therefore \frac{\delta E_0}{E_i} = \frac{\delta R}{4R}$$

Note that $\frac{\delta R}{R} = GF\varepsilon$

$$\Rightarrow \frac{\delta E_0}{E_i} = \frac{GF\varepsilon}{4}$$

4. Multiple gauge Bridge

Use of morethan one active strain gauges

Advantage: (i) increase the resolution
(ii) compensate unwanted effects

Assume all the four bridges are active (full bridge)

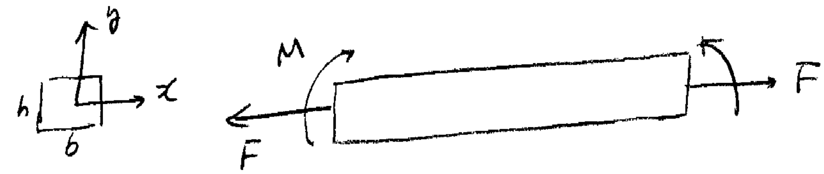
$$\delta E_0 = \sum_{i=1}^{4} \frac{\partial E_0}{\partial R_i} \delta R_i$$

For $R_1 = R_2 = R_3 = R_4$, and $GF_i \equiv GF$ $(i = 1, \cdots 4)$

$$\delta E_0 = \frac{1}{4} GF \left(\varepsilon_1 - \varepsilon_2 + \varepsilon_4 - \varepsilon_3 \right)$$

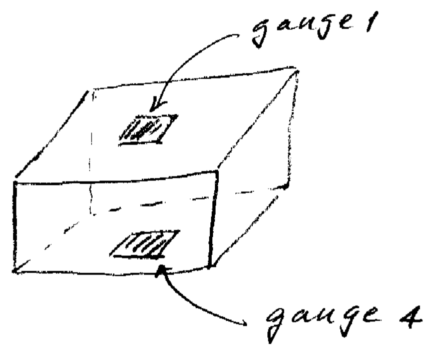
A: remove certain strain component

$$\sigma_x = \frac{-12My}{bh^3}$$



a. remove bending component

using half Bridge. 1 and 4 are active



$$\frac{\delta E_o}{E_i} = \frac{GF}{4}(\varepsilon_1 + \varepsilon_4)$$

$$\varepsilon_1 = \varepsilon_{a1} + \varepsilon_{b1}$$

$$\varepsilon_4 = \varepsilon_{a4} + \varepsilon_{b4} = \varepsilon_{a1} - \varepsilon_{b1}$$

$$\therefore \quad \varepsilon_1 + \varepsilon_4 = 2\varepsilon_{a1} = 2\varepsilon_a$$

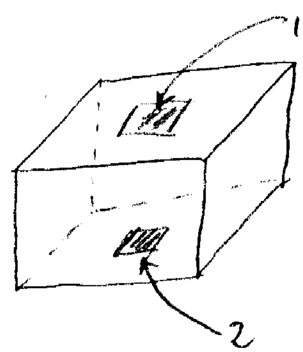
$$\frac{\delta E_o}{E_i} = \frac{GF}{2}\varepsilon_a$$

b. Remove the axial strain due to F

using half Bridge and Bridges 1 and 2 are active

$$\varepsilon_1 = \varepsilon_{a1} + \varepsilon_{b1}$$

$$\varepsilon_2 = \varepsilon_{a2} + \varepsilon_{b2} = \varepsilon_{a1} - \varepsilon_{b1}$$



$$\frac{\delta E_o}{E_i} = \frac{GF}{4}(\varepsilon_1 - \varepsilon_2)$$

$$= \frac{GF}{2}\varepsilon_{b1} = \frac{GF}{2}\varepsilon_b$$

C. Remore the temperature effect

using half Bridge, Bridge 1 and 2 are active

$$\varepsilon_1 = \varepsilon_a + \varepsilon_T$$

$$\varepsilon_2 = \varepsilon_T$$

$$\frac{\delta E_o}{E_i} = \frac{GF}{4}(\varepsilon_1 - \varepsilon_2) = \frac{GF}{4}\varepsilon_a$$