

Process Improvement through Design and Analysis of Experiments

Lecture XV(b)
[Chapter 11 in textbook]

Factorial Designs

Terminology

Factors: *Tire Make* *Temp.*
Levels: A B C 300° 350° 400°

Factorial Design

**All Levels of Each Factor are Combined
with all Levels of Every Other Factor**

		<i>First Factor</i>		
		A	B	C
<i>Second Factor</i>	1	--	--	--
	2	--	--	--
	3	--	--	--

Example of Factorial Design

		<i>Training Device</i>		
		<u>A₁</u>	<u>A₂</u>	<u>A₃</u>
<i>Content B₁</i>	48	28	7	
	58	33	15	

<i>Content B₂</i>	62	14	9	
	54	10	6	

Two - Way Factorial Design

		<i>First Factor</i>		
		A ₁	A ₂	A ₃
<i>Second Factor</i>	B ₁	-	-	-
	B ₂	-	-	-
	B ₃	-	-	-

Model

$$y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{ijk}$$

Calculating the Effects

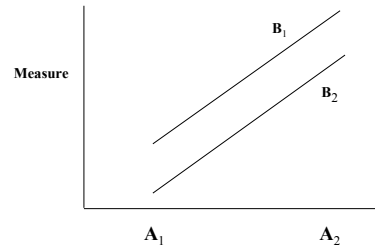
		<i>Training Devices</i>			
		A ₁	A ₂	A ₃	
<i>Content B₁</i>	53.0	30.5	11.0		(31.5)
<i>Content B₂</i>	58.0	12.0	7.5		(25.83)
	(55.5)	(21.25)	(9.25)		

[Grand Mean =28.67]

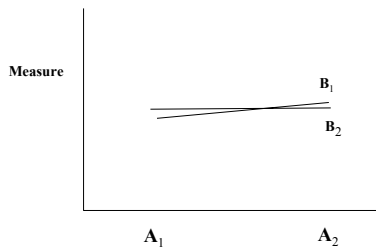
ANOVA Table for a Two-Way Factorial

Source	df	SS	MS	F	α
Devices	2	4608	2304	99.5	< .0001
Contents	1	96	96	4.2	.088
Interaction	2	283	141	6.1	.036
Error	6	139	23		

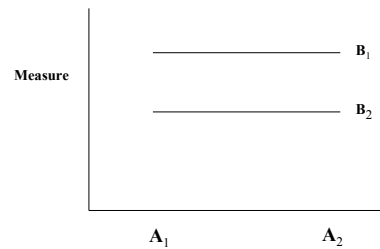
Examples of Two-way Interaction



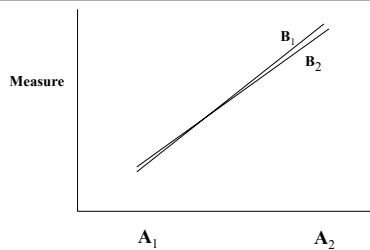
Examples of Two-way Interaction (cont.)



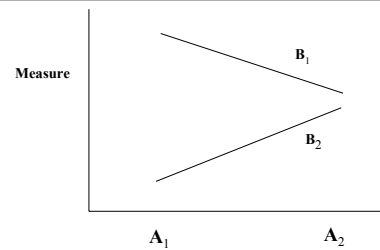
Examples of Two-way Interaction (cont.)



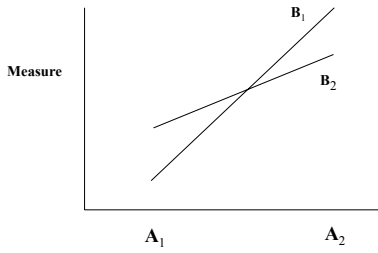
Examples of Two-way Interaction (cont.)



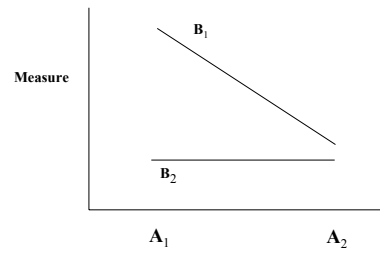
Examples of Two-way Interaction (cont.)



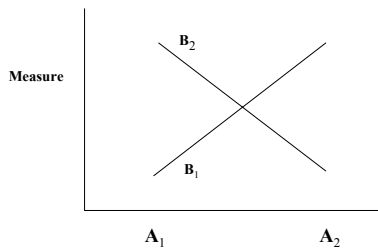
Examples of Two-way Interaction (cont.)



Examples of Two-way Interaction (cont.)



Examples of Two-way Interaction (cont.)



Three Way Analysis of Variance

	A ₁			A ₂		
	B ₁	B ₂	B ₃	B ₁	B ₂	B ₃
C ₁	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃
C ₂	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃
C ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃	S ₁ S ₂ S ₃

Three-Way Factorial Design

Model

$$y_{ijkl} = \mu + A + B + C + AB + AC + BC + ABC + \varepsilon$$

Example of Analysis of Variance Computations

Missile System	Propellant type				Averages
	B ₁	B ₂	B ₃	B ₄	
A ₁	34.0	30.1	29.8	29.0	30.500
A ₂	32.7	32.8	26.7	28.9	
A ₃	32.0	30.2	28.7	27.6	
A ₄	33.2	29.8	28.1	27.8	
A ₅	28.4	27.3	29.7	28.8	28.600
A ₆	29.3	28.9	27.3	29.1	
Averages	31.600	29.850	28.383	28.700	29.592

Missile System	Propellant type				Averages
	B ₁	B ₂	B ₃	B ₄	
A ₁	33.33	31.45	28.25	28.95	30.500
A ₂	32.60	30.00	28.40	27.70	
A ₃	28.85	28.10	28.50	28.95	
Averages	31.600	29.850	28.383	28.700	

Source	SS	df	MS	Observed F	Critical F's (α = .05/.01)
A - Missile Sys	14.52	2	7.26	3.85	3.89 / 6.93
B - Propellant	40.08	3	13.36	10.77	3.49 / 5.95
Interaction	22.17	6	3.70	3.98	3.00
Error	14.91	12	1.24		
Total	91.68	23			

Hierarchical (Nested) Designs

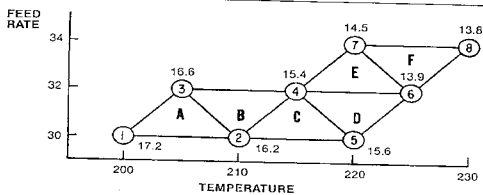
Head	Machine		
	A	B	C
1	6	8	0
	2	9	0
	0	7	5
	8	6	3
2	--	--	--
3	--	--	--
4	3	2	9
	3	1	9
	9	1	6
	8	4	8

Hierarchical (Nested) Designs

Machine	A				B				C			
Head	1	2	3	4	5	6	7	8	9	10	11	12
	6	8	0	3	8	4	6	2	0	4	0	9
	2	9	0	3	9	5	6	1	0	6	1	9
	0	7	5	9	7	2	9	1	5	6	1	6
	8	6	3	8	6	4	5	4	3	5	3	8

[Head 1 is only used on Machine A]

Evolutionary Operations (EVOP)



Confounding

Unintentional

Intentional

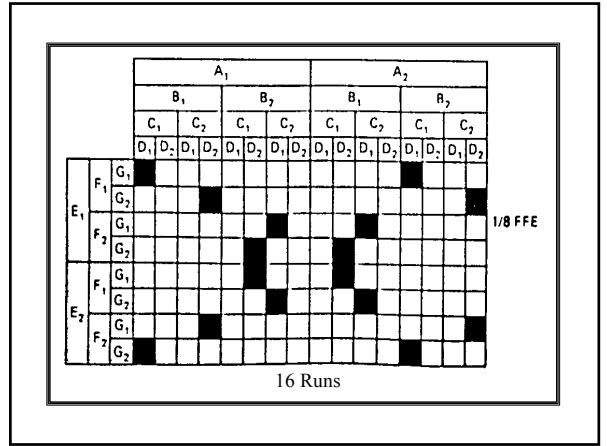
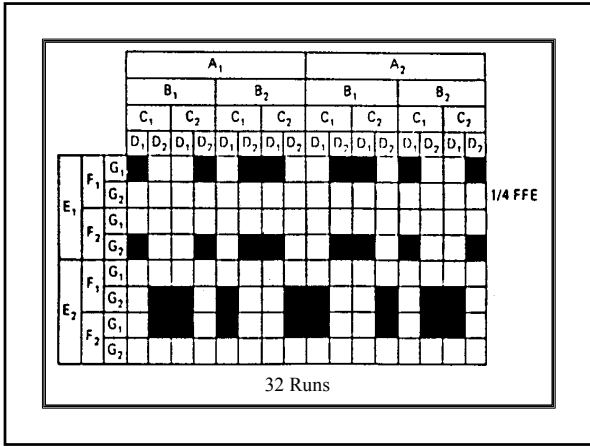
		A ₁				A ₂					
		B ₁		B ₂		B ₁		B ₂			
		C ₁	C ₂	C ₁	C ₂	C ₁	C ₂	C ₁	C ₂		
		D ₁	D ₂	D ₁	D ₂	D ₁	D ₂	D ₁	D ₂		
E ₁	F ₁	G ₁									
	F ₂	G ₂									
E ₂	F ₁	G ₁									
	F ₂	G ₂									

Seven Factor Study - 124 Runs

		A ₁				A ₂					
		B ₁		B ₂		B ₁		B ₂			
		C ₁	C ₂	C ₁	C ₂	C ₁	C ₂	C ₁	C ₂		
		D ₁	D ₂	D ₁	D ₂	D ₁	D ₂	D ₁	D ₂		
E ₁	F ₁	G ₁									
	F ₂	G ₂									
E ₂	F ₁	G ₁									
	F ₂	G ₂									

64 Runs

1/2 FFE



1/8th Fractional Replication

[16 Instead of 124 Runs]

Confounding

A = BCE = DEF = CDG = BFG
AB = CE = FG

1/16th Fractional Replication

[8 Instead of 124 Runs]

Confounding

A = BD = CE = FG = CDF = BEF ...