Process Improvement through Design and Analysis of Experiments

Lecture XV(a)

[Chapter 11 in textbook]

Process Improvement through Design and Analysis of Experiments

Analysis of Variance

(Downtime by Shift)

1st	2nd	3rd
35 min.	**37 min.**	**32 min.**

Analysis of Variance

	Study 1			*Study 2*		
	1st	**2nd**	**3rd**	**1st**	**2nd**	**3rd**
	24	35	19	35	37	32
	33	38	45	35	37	32
	39	46	28	35	37	32
	44	29	36	35	37	32
Means	**35**	**37**	**32**	**35**	**37**	**32**

Analysis of Variance Example

Additive

	A	B	C	D
	193	255	250	233
	238	272	268	242
	220	275	232	228
	225	270	228	225
Means	219	268	242	232
			Grand Mean =	**240**

Statistical Model

The Observed Value Is Equal To:

the Grand Mean

plus the Effect

plus Error

$$y_{ij} = \mu + \text{Treatment Effect}_i + \varepsilon_{ij}$$

"Partitioning" the Variation

Sums of Squares

Total: Sum of (individual data – grand mean) ^{2}

Effect: Sum of (group mean from grand mean) ^{2}

Error: Sum of (individual data from group mean)

"Partitioning" the Variation

Additive

	A	B	C	D
	193	255	250	233
	238	272	268	242
	220	275	232	228
	225	270	228	225
Means	219	268	242	232
		Grand Mean =		**240**

Example Sum of Squares Calculation

SSA = $(219-240)^2 + (268-240)^2 + (242-240)^2 + (232-240)^2$ = **5201**

SSE = $(193-219)^2 + (238-219)^2 + \ldots + (225-232)^2$ = **2438**

SST = $(193-240)^2 + (238-240)^2 + \ldots + (225-240)^2$ = **7639**

Relationship Between Sum of Squares and Variance

Variance = $\Sigma (X_i - \bar{X})^2 / (n - 1)$

= Sum of Squares / Deg. of Freedom

Mean Square = $\Sigma (X_i - \bar{X})^2$ / Deg. of Freedom

Mean Square = Variance (nothing new)

Testing the Hypothesis that there are Differences

<u>Variation due to Effect (MS Additive)</u>
Variation due to Error (MSE)

If this ratio is large, conclude that there is a difference.

If this ratio is not large, can not concluded that there is a difference.

Example Hypothesis Test

MS(Additive) / MSE = 8.53

The Critical Value Depends On How Many Observations (Data) You Have.

*The Statistical Model is the **F-Distribution***

Degrees of Freedom

Additive

	A	B	C	D
	193	255	250	233
	238	272	268	242
	220	275	232	228
	225	270	228	225
Means	219	268	242	232
		Grand Mean =		**240**

Format of the Analysis of Variance Table

Source	df	SS	MS	F	α
Additive	3	5201	1734	8.53	**.0026**
Error	12	2438	203		

Drawing Conclusions Based on an Analysis of Variance

The calculated alpha (α) indicates the probability of observing a difference in the observed means as large or larger, by chance alone, even thought there is no real difference among the population means.

Interpreting Analysis of Variance Results

The results of the ANOVA indicate the probability of at least one mean (e.g., largest) being different from at least one other mean (e.g., smallest).

To evaluate differences among other means, further analyses must be performed. Examples of three tests are:

Duncan-Waller
Schefee Test
Newman-Keuls

Individual Means Comparisons

Additive	*Means*	*Signif.*	
A	**219**	|	
D	**232**	|	
C	**242**	|	|
B	**258**		|

Confounding

Study to test four makes of tires:

[A, B, C and D]

Procedure:

Choose four cars and put one of the tire makes on each car.

The types of cars used are: family sedan, economy, sports and luxury.

Example of Confounding in a Tire Study

Cars

1	**2**	**3**	**4**
A	B	C	D
A	B	C	D
A	B	C	D
A	B	C	D

Are we studying differences in tires or differences in cars?

Completely Randomized Design

Cars

1	2	3	4
C	A	D	A
A	A	C	D
D	B	B	B
D	C	B	C

[Randomized assignment of tires to cars]

Control Confounding by *Blocking*

Cars

1	2	3	4
B	D	A	C
C	C	B	D
A	B	D	B
D	A	C	A

[Each brand of tire is on each car]

Model for Randomized Block Design "Partitioning the Variation"

$$y_{ij} = \mu + \text{Treatment Effect} + \text{Block Effect} + \varepsilon_{ij}$$

Comparison of Randomized Design and Blocked Design

Tire Brand

		A	B	C	D	Means
	1)	17	14	12	13	**14.00**
	2)	14	14	12	11	**12.75**
Car	3)	13	13	10	11	**11.75**
	4)	13	8	9	9	**9.75**
Means		**14.25**	**12.25**	**10.75**	**11.00**	**12.063**

ANOVA Table for Randomized Design

Source	df	SS	MS	F	α
Tires	3	30.69	10.23	2.44	>0.05
Error	12	50.25	4.19		
Total	15	80.94			

[Conclusion: Tires do **Not** make a difference**]**

ANOVA Table for Blocked Design

Source	df	SS	MS	F	α
Tires	3	30.69	10.23	7.85	< 0.01
Blocks	3	38.69	12.90	9.92	<0.01
Error	9	11.56	1.28		
Total	15	**80.94**			

[Conclusion: Tires make a **Large** difference**]**

Note: Both Conclusions are from the same data!!

Trade-off for Blocked Designs

Reduction in the potential for confounding.

Increased efficiency (e.g., power) if the block accounts for some of the variation

Loss of power if the block does not account for very much of the variation.

Latin Square Design
(Blocking in Two Variables)

	Cars			
	1	**2**	**3**	**4**
Position on Car	A	B	C	D
	B	C	D	A
	C	D	A	B
	D	A	B	C

$$y_{ij} = \mu + \text{Treatment Effect} + \text{Row Effect} + \text{Column Effect} + \varepsilon_{ij}$$

ANOVA Table for Latin Square Design

Source	df	SS	MS	F	α
Tires	3	30.69	10.23	11.36	< 0.05
Cars	3	38.69	12.90	14.33	< 0.05
Positions	3	6.19	2.06	2.29	> 0.05
Error	6	5.37	0.90		
Total	15	**80.94**			

[**Conclusion**: Adding ***position*** does not increase the efficiency and power of the study.]

Balanced Incomplete Block Designs

Blocks	A	*B*	*C*	*D*
1)	17	--	12	13
2)	--	14	12	11
3)	13	13	10	--
4)	13	8	--	9