Hypothesis Testing in Quality Improvement

Lecture XIV

[Chapter 11 in textbook]

Sampling Distribution of the Mean

[Central Limit Theorem]

For any population distribution,
as the sample size (n) increases,
the sampling distribution of the mean
approaches normality with the
standard error of the mean = $\sigma/\sqrt{n}$

Example of Cards from 1 to 10

Probability of getting a 1 by chance?
[1/10 or 0.10]

Probability of getting an **average** of 1 when you pick two?
[1/10 * 1/10 or 0.01]

Probability of getting an **average** of 1 when you pick four?
[1/10 * 1/10 * 1/10 * 1/10 or $(1/10)^4$ or 0.0001]

Example of Cards from 1 to 10
(continued)

Probability of getting a 5 by chance?
[same as probability of getting a 1]

Probability of getting an **average** of 5 when you pick two?
[5&5 or 4&6 or 3&7, etc.]

Probability of getting an **average** of 1 when you pick four?
[many ways]

Example of Cards from 1 to 10
(continued)

The "sampling" distribution of the mean gets narrower.
[by the ratio of $1/\sqrt{n}$]

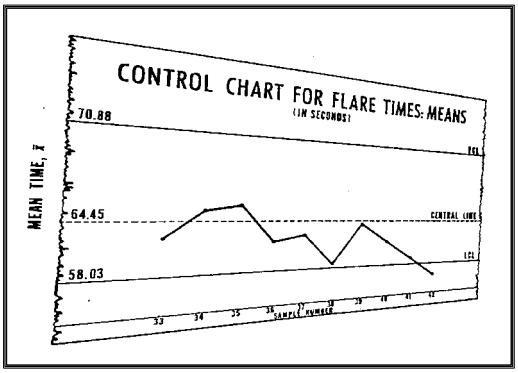
Distribution has a "hump" in the center
[becoming "bell shaped"]

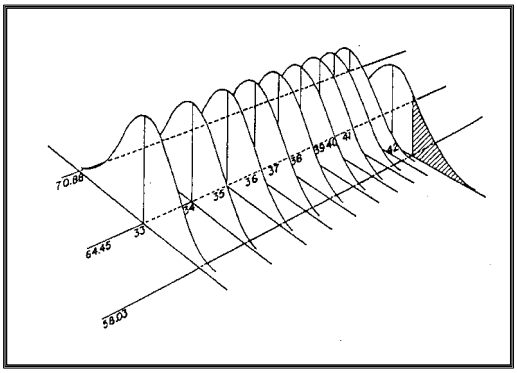
Three "kinds" of Distributions

Population (what you are making judgements about)

Sample (what you are observing)

Sampling (the distribution of the means)
[used in hypothesis testing]





95% Confidence Limits for the Mean

Upper Confidence Limit =Average + 2* sd/√n

Lower Confidence Limit =Average – 2 * sd/√n

95% Confident that the true mean is between these two values.

Confidence and Control Limits

***Confidence Limits** [can be fixed in time]*

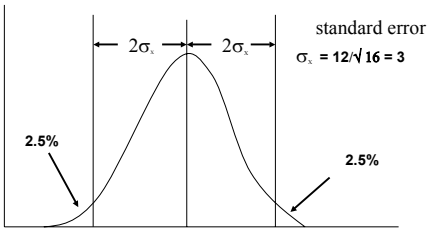
***Control Limits** [evaluated over time]*

Example 95% Confidence Interval

Machine Down Time

Randomly picked 16 days of data.
Observed average down time was 47 minutes
Standard deviation known to be 12 minutes

Example of 95% Confidence Interval



Example of 95% Confidence Interval

Machine Down Time

Randomly picked 16 days of data.

Observed average down time was 47 minutes

Standard deviation known to be 12 minutes

[From the normal distribution there are approximately 2.5% of beyond 2 standard deviations away from the mean]

Lower Conf. Interval = $47 - [(2) * (12/\sqrt{16})] = 47 - 6 = 41$

Upper Conf. Interval = $47 + [(2) * (12/\sqrt{16})] = 47 + 6 = 53$

Determining Sample Size for a Confidence Interval

Machine Down Time Example

Standard deviation known to be 12 minutes

How many days data would you need for a confidence interval of ± 4 minutes (instead of 6, as before)?

$CI = Mean \pm sd/\sqrt{n}$ ["$sd/\sqrt{n}$" is called the "standard error of the mean"]

$CI = Mean \pm 4$ min.

$4 = 2 * sd/\sqrt{n}$

$4 = 2 * 12/\sqrt{n}$ [because we know the standard deviation]

$n = (2*12/4)^2$ [through algebra]

$n = 36$

Stages of the Development of a Law

Conjecture
Hypothetical Construct
Theory
Converging Operations
Law

Hypothesis Testing

Evaluating the probability that the observed difference occurred by chance alone, when there is really no difference.

Types of Errors in Statistical Analysis

Conclude that there ***is*** a difference when there ***is not*** (α or false alarm)

Conclude there ***is not*** a difference when, in fact, there ***is*** (β or miss)

Goal of Statistical Analysis

Conclude that there ***is*** a difference when, in fact, there ***is***

Power of the Test

Critical Region for Test

Point(s) beyond which it can be concluded that the chances are very low that the difference observed would occur, by chance alone, if there is no true difference.

Example of Hypothesis Test

Machine Down Time

Randomly picked 16 days data.
Observed average down time was ***53*** minutes
Standard deviation known to be 12 minutes

*Can you conclude that the true down time is more than **47***

Example of Hypothesis Test

Machine Down Time

Randomly picked 16 days data.
Observed average down time was ***53*** minutes
Standard deviation known to be 12 minutes

Difference between the hypothesized (47) and the observed (54)

is 6 minutes

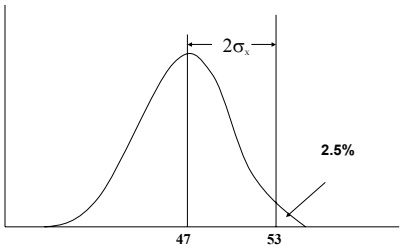
Hypothesis Test Machine Down Time

Randomly picked 16 days data.
Observed average down time was ***53*** minutes
Standard deviation known to be 12 minutes

The standard error is $12/\sqrt{16}$ or 3

An observed difference of 6 minutes is 2 standard errors (6/3)

Hypothesis Test Machine Down Time

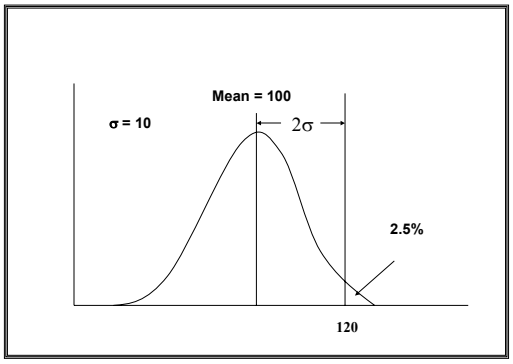


Hypothesis Test Machine Down Time

Randomly picked 16 days data.
Observed average down time was ***53*** minutes
Standard deviation known to be 12 minutes
The standard error is $12/\sqrt{16}$ or 3
An observed difference of 6 minutes is 2 standard errors (6/3)
Using the normal distribution, the probability of being 2 or more standard errors from the mean is approximately 2.5%

Result: there is only a 2.5% chance that you would observe a difference of 6 by chance alone if there were really no difference.

Conclusion: there is a difference.



Importance of the Power of the Test

If the true value of the down time for the machine is **56**, what is the power of the test we just conducted?

Using both False Alarm and Power

Historical Mean: 47
Mean where an effect occurs: 56
Standard deviation: 12

Chances of a false alarm (α) = 0.05
Power of the test = 0.95

Control Chart for Means

Test the hypothesis that the mean has shifted
[3σ limits; α = .0027]

Power is often very low

Testing for Differences

Between Two Means

Z - test [σ known or assumed]
t - test [σ not known]

Among More than Two Means

Analysis of Variance