

HMWK 1; DIFFERENTIAL GEOMETRY, FALL 2009

NAME :

Due on Monday, Sept. 7 2009
Justify all answers in detail!!!

Work out problems 1-5, on page 9 and problems 1,2,3,4(a)(b) on page 24.

Additional problem 1. Let $a_1, \dots, a_n > 0$ be positive scalars.

- (a) Show that $|x|_A = \sqrt{\sum_{i=1}^n a_i x_i^2}$ defines a norm on \mathbb{R}^n .
- (b) Does $|x|_A$ arise out of a particular dot-product? (i.e. $|x|_A^2 = \langle x, x \rangle_A$ for some dot product $\langle \cdot, \cdot \rangle_A$).
- (c) Show through an example that if at least one a_i is nonpositive then $|x|_A$ fails to be a norm.

Problem 2. Describe, in terms of action on some sample vectors, the action of the following linear transformations of \mathbb{R}^2 . (an additional graphic description, with sketches is preferred).

$$\begin{aligned}x \in \mathbb{R}^2 &\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot x \\x \in \mathbb{R}^2 &\rightarrow \begin{pmatrix} & 1 \\ -1 & 0 \end{pmatrix} \cdot x \\x \in \mathbb{R}^2 &\rightarrow \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot x \\x \in \mathbb{R}^2 &\rightarrow \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \cdot x \quad \text{as } k \rightarrow \infty\end{aligned}$$