

Sharp Friedrichs -Knapp-Stein inequalities and applications

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Abstract

The classical Friedrichs identity states that for $u \in C_0^\infty(\mathbb{R}^n)$ we have

$$\int_{\mathbb{R}^n} |D^2 u|^2 dx = \int_{\mathbb{R}^n} |\Delta u|^2 dx.$$

From this inequality we immediately get $W^{2,2}$ -estimates for solutions of $\Delta u = f$ and also for solutions of measurable perturbations of the form $\sum_{ij} a_{ij}(x)u_{ij}(x) = f(x)$, when the matrix $A = (a_{ij})$ is closed to the identity in sense made precise by Cordes. In this talk we explore extensions of the Friedrichs identity in the form of sharp inequalities

$$\int_X |\mathfrak{X}^2 u|^2 dx \leq C \int_X |\Delta_{\mathfrak{X}} u|^2 dx.$$

where X is a Riemannian manifold, a nilpotent Lie group, and a CR manifold.

This is joint work with Andràs Domokos (PAMS 133, 2005) and Sagun Chanillo (2007 preprint.)