Carnot-Carathéodory inf-convolution and limiting behavior of solutions of subelliptic heat kernels.

Abstract: It is well-known that the limiting behavior, as $\varepsilon \to 0^+$, of the solutions of

$$\begin{cases} w_t^\varepsilon - \varepsilon \Delta w^\varepsilon = 0, & x \in \mathbb{R}^n,\ t > 0, \\ w^\varepsilon(0,x) = \mathrm{e}^{-\frac{g(x)}{2\varepsilon}}, & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

is described by the Hamilton-Jacobi-Cauchy problem

$$\begin{cases} u_t + \dfrac{1}{2}|Du|^2 = 0, & x \in \mathbb{R}^n,\ t > 0, \\ u(0,x) = g(x), & x \in \mathbb{R}^n, \end{cases} \tag{2}$$

More precisely, if $g : \mathbb{R}^n \to \mathbb{R}$ is a bounded and continuous function, the logarithmic transform of w^ε, i.e. $u^\varepsilon = -2\varepsilon \log w^\varepsilon$, converges, locally uniformly, as $\varepsilon \to 0^+$, to the unique viscosity solution u of (2). One way of proving this is to use both the representation of w^ε as the integral convolution and the Hopf-Lax representation of the viscosity solution of (2) as the (euclidean) inf-convolution

$$g_t(x) = \inf_{y \in \mathbb{R}^n} \left[g(y) + \frac{|x-y|^2}{2t} \right],$$

and to apply the Large Deviation Principle in order to establish the validity of the limiting relation

$$\lim_{\varepsilon \to 0^+} -2\varepsilon \log w^\varepsilon = u.$$

The aim of this talk is to generalize the procedure described above in order to analyze the limiting behavior of some subelliptic diffusion equations in term of the Carnot-Carathéodory inf-convolutions, given a new proof covering as the Hörmander case as the already known euclidean case, using measure theory methods instead of the probability techniques.