ELEG 3143 Probability and Stochastic Processes
MIDTERM EXAM I
February 28, 2014  9:40am - 10:30am

Name: ________________________________ UID: ________________________________

• This is a closed book and notes exam.

• Calculators are allowed.

• To receive full credits, please show the procedure, equations, and calculations clearly and concisely.

<table>
<thead>
<tr>
<th>Question</th>
<th>P1 (18%)</th>
<th>P2 (16%)</th>
<th>P3 (26%)</th>
<th>P4 (20%)</th>
<th>P5 (20%)</th>
<th>Total</th>
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<tr>
<td>Grade</td>
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1. Computer programs are classified by the length of the source code and by the execution time. Programs with more than 150 lines in the source code are big ($B$). Programs with $\leq 150$ lines are little ($L$). Fast programs ($F$) run in less than 0.1 seconds. Slow programs ($W$) require at least 0.1 seconds. Monitor a program executed by a computer. Observe the length of the source code and the run time. The probability model for this experiment contains the following information: $P[LF] = 0.5$, $P[BF] = 0.2$, and $P[BW] = 0.2$. Please calculate the following probabilities:

(1) $P[W]$ (6 points)
(2) $P[B]$ (6 points)
(3) $P[W \cup B]$ (6 points)

**Solution:**

The probabilities specified in the problem can be represented in the following table:

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>L</th>
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<tbody>
<tr>
<td>$F$</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>$W$</td>
<td>0.2</td>
<td></td>
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</tbody>
</table>

Thus, $P[WL] = 0.1$.

(1) $P[W] = P[WB] + P[WL] = 0.2 + 0.1 = 0.3$.
(2) $P[B] = P[BF] + P[BW] = 0.2 + 0.2 = 0.4$.
(3) $P[W \cup B] = P[W] + P[B] - P[WB] = 0.3 + 0.4 - 0.2 = 0.5$. 
2. At the end of regulation time, a basketball team is trailing by one point and a player goes to the line for two free throws. If the player makes exactly one free throw, the game goes into overtime. The probability that the first free throw is good is $1/2$. If the first attempt is good, the player relaxes and the second attempt is good with probability $3/5$. However, if the player misses the first attempt, the added pressure reduces the success probability to $2/5$.

1) Draw the tree diagram for this experiment. (8 points)

2) Calculate the probability that the game goes into overtime. (8 points)

Solution:
Let $G_i$ = the $i$-th throw is good, $M_i$ = the $i$-th throw is missed, $i = 1, 2$. Then,

$$P[\text{overtime}] = P[G_1M_2] + P[M_1G_2]$$
$$= P[M_2|G_1]P[G_1] + P[G_2|M_1]P[G_2]$$
$$= 2/5 \times 1/2 + 2/5 \times 1/2 = 2/5$$
3. Before going on vacation for a week, you ask your friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. Even with proper watering, it has a 20 percent chance of dying. And the probability that your friend will forget to water it is 30 percent.

(1) What is the chance that your plant will survive the week? (10 points)
(2) If it is dead when you return, what is the chance that your friend forgot to water it? (10 points)
(3) If your friend forgot to water it, what is the chance that it will be dead when you return? (6 points)

Solution:
Let $W =$ watering, $S =$ survive. Then, $P[W] = 0.7$, $P[W^c] = 0.3$. $P[S^c|W^c] = 0.9$, $P[S^c|W] = 0.2$.

(1)
\[
= (1 - P[S^c|W])P[W] + (1 - P[S^c|W^c])P[W^c] \\
= (1 - 0.2)0.7 + (1 - 0.9)0.3 = 0.59
\]

(2)
\[
P[W^c|S^c] = \frac{P[W^cS^c]}{P[S^c]} = \frac{P[S^c|W^c]P[W^c]}{1 - P[S]} = \frac{0.9 \times 0.3}{1 - 0.59} = \frac{27}{41}
\]

(3)
\[
P[S^c|W^c] = 0.9
\]
4. To communicate one bit of information reliably, cellular phones transmit the same binary symbol five times. Thus information “zero” is transmitted as 00000 and “one” is transmitted as 11111. Each binary symbol may get flipped during the transmission. The receiver detects the correct information if four or more binary symbols are received correctly. If there are two or three binary symbols are received correctly, i.e. two or three zeros are received, a deletion (event $D$) occurs. If only one or no binary symbol is received correctly, a decoding error (event $E$) occurs. Assume the binary symbol error probability is $q = 0.1$. Please calculate the following probabilities:

(1) $P[E]$ (10 points)
(2) $P[D]$ (10 points)

**Solution:**

Using $S_{k,5}$ to denote the event of $k$ successes in the five trials, then the probability $k$ bits are decoded successfully at the receiver is

$$P[S_{k,5}] = \binom{5}{k} p^k (1 - p)^{5-k}, \quad k = 0, 1, \ldots, 5.$$ where $p = 1 - q = 0.9$.

(1)

$$P[E] = P[S_{1,5}] + P[S_{0,5}] = 5p(1 - p)^4 + (1 - p)^5 = 0.00046.$$  

(2)

$$P[D] = P[S_{3,5}] + P[S_{2,5}] = 10p^3(1 - p)^2 + 10p^2(1 - p)^3 = 0.081.$$
5. An operation has five components. Each component has a success probability $p = 0.8$, independent of any other component. The operation is successful if either part A or part B works. Part A is successful if components 1, 2 and 3 all work. Part B is successful if components 4 and 5 both work. The above operation is shown in the following diagram.

Please calculate the probability $P[W]$ that the entire operation is successful. (20 points)

**Solution:**
Since $W_1, W_2, W_3$ are connected in series, we have


The operation is successful if either part A or part B works. Thus,