

Optical switching in arrays of quantum dots with dipole-dipole interactions

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(Dated: September 13, 2006)

We explore the possibility of using pairs of quantum dots coupled by the dipole-dipole interaction as effective three- or four-level systems whose transmission for an optical beam at some frequency may be switched on or off using a second optical beam (electromagnetically-induced transparency). We conclude that the characteristic interaction strengths and decay rates should allow for a demonstration of this effect in MBE-grown bilayer InAs/GaAs quantum dot structures.

PACS numbers: 78.67.Hc, 71.35.Gg, 42.50.Gy

I. INTRODUCTION

There has been recently much interest in the study of systems of pairs of coupled quantum dots¹⁻⁷, where the coupling can be due to either tunneling, or the electric dipole interaction between excitons. In quantum wells, it has been shown both theoretically and experimentally that the coupling resulting from tunneling can be used for optical switching⁸, via a mechanism similar to Fano interference. In this paper, we point out that for quantum dots coupled by the dipole interaction^{2,9,10} one may obtain effective three- or four-level systems where the techniques of Electromagnetically-Induced Transparency (EIT) could also be used for optical switching (see Ref. 5 for a proposal of coherent population transfer, an effect closely related to EIT, in tunneling-coupled quantum dots, and Ref. 11 for a demonstration of EIT in a single quantum well via biexciton coherence).

In the remainder of this Introduction we discuss the order of magnitude of the expected level splittings, and the various level arrangements that may result. In the next section we develop a simple analytical model showing the basic features of the switching scheme, and in the section following that we compare this model with more complete numerical calculations.

The dipole moment of a dot of size a may be taken to be of the order of ea and the dipole-dipole interaction energy between two dots separated a distance r is then of the order of $e^2a^2/4\pi\epsilon_0r^3$. As an example, if $a = 4$ nm and $r = 20$ nm, this energy is of the order of 2.9 meV.

Two mechanisms for dipole-dipole coupling have been discussed in the literature (see Ref. 7 for more details). The first, Förster energy transfer, involves the exchange of a virtual photon between dots separated by a distance smaller than the wavelength of light. This can be represented by a term in the Hamiltonian of the form

$$V_F = \hbar d \left(|eg\rangle\langle ge| + |ge\rangle\langle eg| \right), \quad (1)$$

where $|eg\rangle$ (resp. $|ge\rangle$) represents the state in which there is an exciton in the first (resp. second) dot, and $\hbar d$ is a constant of the order of magnitude discussed in the previous paragraph.

The second mechanism involves a direct, static (Coulomb) interaction between the dipole moments of

the excited dots, and can be represented by a term in the Hamiltonian of the form

$$V_D = \hbar d' |ee\rangle\langle ee|. \quad (2)$$

The result of this term is to make the energy of the biexciton state $|ee\rangle$ different from just the sum of the single exciton states $|eg\rangle$ and $|ge\rangle$.

As we shall show below, either term (1) or (2) can, under the appropriate circumstances, result in a level structure useful for optical switching. For clarity, we show this first analytically considering each term separately, and the simple (but not very realistic) situation in which the single exciton states are degenerate in the absence of the interaction. In a more realistic scenario, the two dots will have different sizes and the states $|eg\rangle$ and $|ge\rangle$ will not be degenerate even in the absence of the dipole-dipole interaction, but here again the addition of either term (1) or (2) results in a level structure in which optical switching is possible. The main requirement in all these cases is that the strength of the coupling, d or d' (in frequency units) be larger than the level widths γ , as we shall see below analytically, and also explore later numerically.

II. BASIC HAMILTONIAN AND COUPLING APPROACHES

A. Switching for identical dots

We adopt the following basic Hamiltonian, involving the lowering operators $\sigma_i = |g_i\rangle\langle e_i|$, $i = a, b$, for each dot ($|g\rangle$ stands for the ground state and $|e\rangle$ for the excited state), their Hermitian conjugates σ_i^\dagger , and two external fields represented by the Rabi frequencies Ω_p (for “probe”) and Ω_c (for “coupling”), in a suitable interaction picture:

$$H = \frac{\hbar}{2} \left(\Omega_p e^{-i\Delta_p t} + \Omega_c e^{-i\Delta_c t} \right) \left(\sigma_a^\dagger + \sigma_b^\dagger \right) + H.c. \\ + \hbar d \left(\sigma_a \sigma_b^\dagger + \sigma_a^\dagger \sigma_b \right) + \hbar d' |ee\rangle\langle ee| \quad (3)$$

Here the two dots have been assumed to be identical and to couple identically to the two fields, an assumption that

will be relaxed later on. The detunings are therefore defined relative to the energy $\hbar\omega_0$ of the single exciton state(s): $\Delta_p = \omega_p - \omega_0$, $\Delta_c = \omega_c - \omega_0$.

1. Pure Förster coupling case

Consider first the case where the term $d' = 0$, i.e., we have pure Förster interaction coupling. We may carry out a partial diagonalization of the Hamiltonian (3) by replacing the states $|eg\rangle$ and $|ge\rangle$ by the symmetric and antisymmetric combinations $|+\rangle$ and $|-\rangle$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|eg\rangle \pm |ge\rangle) \quad (4)$$

which are eigenstates of the d term with eigenvalues $\pm\hbar d$. The other two basis states, $|ee\rangle$ and $|gg\rangle$, are also eigenstates of this term with eigenvalue 0. This allows us to move to another interaction picture where the d term is absent and the Hamiltonian is simply

$$H_I = \frac{\hbar}{\sqrt{2}} (\Omega_p e^{-i\Delta_p t} + \Omega_c e^{-i\Delta_c t}) \times (|ee\rangle\langle+| e^{-idt} + |+\rangle\langle gg| e^{idt}) + H.c. \quad (5)$$

In this idealized situation there is no coupling to the “dark state” $|-\rangle$, and the two dots form an effective three-level system. Switching can take place in this system in the following way. Suppose that we tune the “coupling” beam to the $|+\rangle \rightarrow |ee\rangle$ transition and the probe beam to the $|gg\rangle \rightarrow |+\rangle$ transition; this means that we want $\Delta_c + d \sim 0$ and $\Delta_p - d \sim 0$, and therefore we must have $|\Delta_c - d| \sim |\Delta_p + d| \sim 2d$. Assuming that d is much larger than the levels’ widths, we make a rotating-wave approximation and obtain

$$H_I \simeq \hbar\Omega'_p e^{-i(\Delta_p - d)t} |+\rangle\langle gg| + \hbar\Omega'_c e^{-i(\Delta_c + d)t} |ee\rangle\langle+| + H.c. \quad (6)$$

with $\Omega'_p = \Omega_p/\sqrt{2}$, $\Omega'_c = \Omega_c/\sqrt{2}$. This is now a typical 3-level scheme formally analogous to those encountered in EIT. With the dots initially in the state $|gg\rangle$, we can expect to be able to control the absorption of the probe on the $|gg\rangle \rightarrow |+\rangle$ transition through the use of the coupling beam on the “empty” transition $|+\rangle \rightarrow |ee\rangle$.

The main difference with conventional EIT is in the decay rates. If the level $|e\rangle$ decays at a rate γ , then both the state $|ee\rangle$, with two excited dots, and the superradiant state $|+\rangle$ decay at a rate 2γ . In conventional EIT, on the other hand, the decay rates associated with the two transitions often differ by orders of magnitude.

We can introduce decay terms in a quasi-pure state formalism by writing the wavefunction as

$$|\psi(t)\rangle = C_{ee}|ee\rangle + C_+|+\rangle + C_{gg}|gg\rangle \quad (7)$$

and writing equations of motion for the amplitudes

$$\dot{C}_{ee} = -2\gamma C_{ee} - i\Omega'_c C_+ e^{-i\delta_c t} \quad (8a)$$

$$\dot{C}_+ = -2\gamma C_+ - i\Omega'_p C_{gg} e^{-i\delta_p t} - i\Omega'_c C_{ee} e^{i\delta_c t} \quad (8b)$$

$$\dot{C}_{gg} = -i\Omega'_p C_+ e^{i\delta_p t} \quad (8c)$$

with $\delta_p = \Delta_p - d$, $\delta_c = \Delta_c + d$.

The system (8) does not conserve probability because it does not account for the fact that $|ee\rangle$ decays into $|+\rangle$ and $|+\rangle$ decays into $|gg\rangle$; a careful treatment would have to be based on a density matrix description, but the approximate equations (8) may be sufficient as long as the population of the excited states is small. Under those conditions, one may approximate the expectation value of the (complex) dipole moment of the system as

$$\mu\langle\sigma_a + \sigma_b\rangle \simeq \mu\sqrt{2}C_{gg}^* C_+ \simeq \mu\sqrt{2}C_+ \quad (9)$$

where μ is the matrix element of the dipole moment operator in between the states $|e\rangle$ and $|g\rangle$. Here we have first neglected a term proportional to $C_{ee}C_+^*$ (compare Eq. (23) below), and then assumed the depletion of the ground state is small, so $C_{gg} \simeq 1$. The value of C_+ can be estimated from the equations of motion (8), by first writing them as a time-independent system, via the change of variables $\tilde{C}_+ = C_+ e^{i\delta_p t}$, $\tilde{C}_{ee} = C_{ee} e^{i(\delta_c + \delta_p)t}$, with the result

$$\dot{\tilde{C}}_{ee} = -(2\gamma - i(\delta_c + \delta_p))\tilde{C}_{ee} - i\Omega'_c \tilde{C}_+ \quad (10a)$$

$$\dot{\tilde{C}}_+ = -(2\gamma - i\delta_p)\tilde{C}_+ - i\Omega'_p C_{gg} - i\Omega'_c \tilde{C}_{ee} \quad (10b)$$

$$\dot{C}_{gg} = -i\Omega'_p \tilde{C}_+ \quad (10c)$$

then solving (10a) in the steady state, substituting in (10b), and solving that equation in steady state also, with the assumption that $C_{gg} = 1$. The result for C_+ is correct to all orders in the coupling field Ω_c and to first order in the probe field Ω_p . Substituting in (9), we obtain the complex susceptibility $\chi = (2\mu^2/\epsilon_0\hbar\Omega_p)\langle\sigma_a + \sigma_b\rangle$ in the form

$$\chi = \chi_0 \left[1 - i\frac{\delta_p}{2\gamma} + \frac{\Omega_c^2/8\gamma^2}{1 - i(\delta_p + \delta_c)/2\gamma} \right]^{-1} \quad (11)$$

where $\chi_0 = -i\mu^2/\hbar\epsilon_0\gamma$. Equation (11) is of the same form as standard EIT expressions, such as, e.g., Eq. (3) of Ref. 13; the difference, as already indicated, is in the decay rates. Figure 1 shows the resulting absorption profile, for $\Omega_c/2\gamma = 0, 2$ and 4, as a function of the probe detuning $\delta_p/2\gamma$, for $\delta_c = 0$.

2. Pure static (Coulomb) coupling case

Consider next the case where the Förster coupling term d is negligible and instead we have $d' \neq 0$ in Eq. (3). We can still use the transformation to the symmetric and antisymmetric combinations $|+\rangle$ and $|-\rangle$, as in Eq. (5),

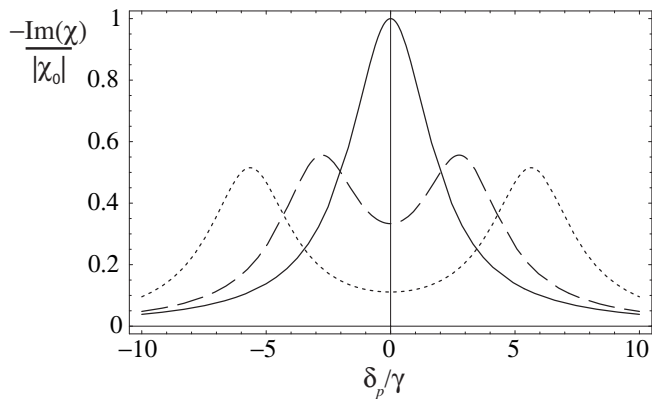


FIG. 1: Absorption rate (imaginary part of the susceptibility (11)) as a function of the probe detuning δ_p/γ , for $\delta_c = 0$ and $\Omega_c/2\gamma = 0$ (solid line), 2 (dashed) and 4 (dotted).

only with $d = 0$; moreover, the term $\hbar d' |ee\rangle\langle ee|$ is now easily accounted for in an appropriate interaction picture just by inserting a factor $e^{id't}$ multiplying the ket $|ee\rangle$ in that equation, with the result

$$H_I = \frac{\hbar}{\sqrt{2}} (\Omega_p e^{-i\Delta_p t} + \Omega_c e^{-i\Delta_c t}) \times (|ee\rangle\langle +|e^{id't} + |+\rangle\langle gg|) + H.c. \quad (12)$$

Thus, we have again an effective three-level system, and if $d' > \gamma$ we can make again the rotating-wave approximation, now requiring that $\Delta_c - d' = \delta_c \sim 0$ and $\Delta_p = \delta_p \sim 0$ (which implies $|\Delta_c| \sim |\Delta_p - d'| \sim d'$). The result is formally equivalent to (6):

$$H_I \simeq \hbar\Omega'_p e^{-i\delta_p t} |+\rangle\langle gg| + \hbar\Omega'_c e^{-i\delta_c t} |ee\rangle\langle +| \quad (13)$$

and thus we would obtain again the result (11) for the susceptibility.

Clearly, in the general case with both d and d' nonzero, the procedure sketched in this subsection will lead to the result (11) if the detunings δ_c and δ_p are defined as

$$\begin{aligned} \delta_c &= \Delta_c + d - d' \\ \delta_p &= \Delta_p - d \end{aligned} \quad (14)$$

or equivalently, the laser frequencies are tuned to $\omega_c \simeq \omega_0 + d' - d$, $\omega_p \simeq \omega_0 + d$.

B. Switching with nonidentical dots

In general, the dots will be different and will couple differently to the field, which means that there will be a small probability of coupling to the “dark,” antisymmetric state $|-\rangle$. More importantly, the excitation energies of the two dots may be different, so instead of the single frequency ω_0 introduced in the previous section, one may

now have to deal with a basic Hamiltonian (excluding the laser fields) of the form

$$H_0 = \hbar\omega_a |eg\rangle\langle eg| + \hbar\omega_b |ge\rangle\langle ge| + \hbar d (|eg\rangle\langle ge| + |ge\rangle\langle eg|) + \hbar(\omega_a + \omega_b + d') |ee\rangle\langle ee| \quad (15)$$

As discussed in Ref. 7, the eigenvalues of this Hamiltonian are $E_0 = 0$, with eigenvector $|gg\rangle$, $E_3 = \hbar(\omega_a + \omega_b + d')$, with eigenvector $|ee\rangle$, and

$$\begin{aligned} E_1 &= \hbar \frac{\omega_a + \omega_b}{2} - \frac{\hbar}{2} \sqrt{(\omega_a - \omega_b)^2 + 4d^2} \\ E_2 &= \hbar \frac{\omega_a + \omega_b}{2} + \frac{\hbar}{2} \sqrt{(\omega_a - \omega_b)^2 + 4d^2} \end{aligned} \quad (16)$$

whose eigenvectors, $|1\rangle$ and $|2\rangle$, are linear combinations of $|eg\rangle$ and $|ge\rangle$.

One can envision two basic limits. In one, the noninteracting energy difference $|\omega_a - \omega_b|$ is smaller than the level width γ , in which case one expects things to work out qualitatively the same as in the previous subsection, provided at least one of either d or d' is greater than γ . In the other limit, $|\omega_a - \omega_b| > \gamma$, and one has two well-separated “middle” levels, $|1\rangle$ and $|2\rangle$, with energies given by Eq. (16).

Even in this limit, however, the dipole-dipole couplings d and d' may still result in a level structure that allows optical switching, in essentially two different ways. First, if $d \gg (\omega_a - \omega_b)/2$, it can be seen from Eq. (16) that the eigenvector $|1\rangle$ is essentially the same as the antisymmetric state $|-\rangle$, and therefore it will be largely decoupled from the external fields, allowing the kind of switching discussed in Section 2.A.1 to be carried out in the ladder-like manifold $\{|0\rangle, |2\rangle, |3\rangle\}$.

Second, in the (possibly more realistic; see Ref. 9) case in which d is smaller than, or of the order of, the asymmetry $(\omega_a - \omega_b)/2$, as long as $d' > \gamma$ one still has a set of transitions that can be used for optical switching. The important point is that, for nonzero d' , all of the four possible transitions have distinct frequencies:

$$\begin{aligned} |0\rangle \rightarrow |1\rangle &: E_1 \\ |1\rangle \rightarrow |3\rangle &: E_3 - E_1 = \hbar d' + E_2 \\ |0\rangle \rightarrow |2\rangle &: E_2 \\ |2\rangle \rightarrow |3\rangle &: E_3 - E_2 = \hbar d' + E_1 \end{aligned} \quad (17)$$

Barring, therefore, an accidental degeneracy (in which, say, $E_2 = \hbar d' + E_1$), one could simply tune the probe beam to the $|0\rangle \rightarrow |2\rangle$ transition, and the coupling beam to the $|2\rangle \rightarrow |3\rangle$ transition, and carry out the EIT-like optical switching described in the previous section, as if the state $|1\rangle$ was not there. (Using state $|2\rangle$ as an intermediate level is preferable to attempting to use state $|1\rangle$, since the latter’s coupling to the fields is likely to be much weaker.)

Of course, in practice, and for finite splitting and widths, there will be some amount of cross-talking between the various modes, and some probability to populate the state $|1\rangle$; also, in the schemes that rely on the

state $|1\rangle$ (or $|-\rangle$) being “dark,” this will not be exactly the case if the dots are not identical and have different coupling strengths to the field. The effects of these deviations from the simple models presented so far are studied numerically in the following Section.

III. NUMERICAL CALCULATIONS

A. Basic equations

The result (11) in the previous section involves several approximations. The most important of these is the neglecting of off-resonant transitions, i.e., the “rotating-wave approximation” that allowed us to replace the Hamiltonian (5) by the simpler (6). In addition to this, decay and dephasing processes have been treated only at the level of the state-vector amplitudes (in Eqs. (8) and (10)), which is also not really consistent. The purpose of the numerical calculations in this section is to remove these approximations and present a more realistic treatment of the two-dot system, based on density-matrix equations of motion.

The standard master equation for radiative decay of a two-level system is of the form (see, e.g., Scully and Zubairy¹⁴, Eq. (8.2.8))

$$(\dot{\rho})_{\text{rad}} = -\gamma(\sigma^\dagger\sigma\rho + \rho\sigma^\dagger\sigma - 2\sigma\rho\sigma^\dagger) \quad (18)$$

where σ is the lowering operator, as in Eq. (3). For two identical systems, much closer than one wavelength (so that the effects of their decay on the environment are indistinguishable), a correct master equation is obtained simply by replacing σ in (18) by $\sigma_a + \sigma_b$ (and similarly for σ^\dagger). As indicated in the previous section, however, we may want to consider what happens when the dots are not quite identical, and therefore we shall introduce a parameter, r , to account for the possibility that dot b may couple to the radiation field with a matrix element larger ($r > 1$) or smaller ($r < 1$) than dot a . This parameter will appear in the corresponding Rabi frequencies (see below, Eq. (22)), and, for consistency, it should also appear in the radiative damping equation (18), where we will substitute $\sigma \rightarrow \sigma_a + r\sigma_b$. The result is

$$(\dot{\rho})_{\text{rad}} = - \sum_{i,j=a,b} \gamma_{ij} \left(\sigma_i^\dagger \sigma_j \rho + \rho \sigma_i^\dagger \sigma_j - 2\sigma_i \rho \sigma_j^\dagger \right) \quad (19)$$

with $\gamma_{aa} = \gamma$, $\gamma_{bb} = r^2\gamma$, and $\gamma_{ab} = \gamma_{ba} = r\gamma$.

The above treatment does not account for pure dephasing, which is expected¹⁵ to be much larger than the radiative decay for strongly confined In(Ga)As/GaAs self-organized quantum dots, such as the ones we would want to perform experiments on eventually. A master equation that yields pure dephasing can be written down¹⁶ with the same form as (18), only replacing the lowering operator σ by the Pauli matrix $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$: application of this operator leaves the energy eigenstates

invariant but scrambles the relative phase of their coherent superpositions. Again, however, we want to replace the single-dot operator σ_z by something like $\sigma_{az} + r'\sigma_{bz}$, where r' is a new parameter to allow, heuristically, for some difference between the dots that might lead to different dephasing rates. We take, therefore, the dephasing contribution to the density-matrix equations of motion to be of the form

$$(\dot{\rho})_{\text{deph}} = -\Gamma(\rho - \sigma_{az}\rho\sigma_{az}) - r'^2\Gamma(\rho - \sigma_{bz}\rho\sigma_{bz}) \\ - r'\Gamma(\sigma_{az}\sigma_{bz}\rho + \rho\sigma_{az}\sigma_{bz} - \sigma_{az}\rho\sigma_{bz} - \sigma_{bz}\rho\sigma_{az}) \quad (20)$$

The decay constant Γ is related to the dephasing relaxation time T_2 for a single dot by $2\Gamma = 1/T_2$.

Like Eq. (19), Eq. (20) assumes that the dephasing process is adequately described as “collective decoherence;” that is, we are assuming that the environmental fluctuations that cause dephasing are perfectly correlated at the positions of both dots. If the main cause of dephasing is, for instance, elastic collisions with phonons, this amounts to the assumption that the phonon wavelength is much larger than the separation between the dots, which is probably a good approximation at sufficiently low temperatures. A heuristic way to account for a less-than-perfect correlation would be to multiply the last term in (20) by a “correlation parameter” ξ , where $\xi = 1$ for perfectly correlated, and $\xi = 0$ for totally independent, dephasing. We have, in fact, done calculations both with $\xi = 1$ and $\xi = 0$ and we have found, perhaps somewhat surprisingly, only relatively minor differences between the results in the two cases (at least, in the range of parameters we have explored), so we have decided to just set $\xi = 1$ above, for simplicity.

The total master equation is thus of the form

$$\dot{\rho} = (\dot{\rho})_{\text{rad}} + (\dot{\rho})_{\text{deph}} - \frac{i}{\hbar}[H, \rho] \quad (21)$$

with the Hamiltonian

$$H = H_0 + \frac{\hbar}{2} \left[(\Omega_p e^{-i\omega_p t} + \Omega_c e^{-i\omega_c t}) (\sigma_a^\dagger + r\sigma_b^\dagger) + H.c. \right] \quad (22)$$

where H_0 is given by Eq. (15). The full set of density-matrix equations is written out in the Appendix, in the interaction picture used in the calculations.

To calculate the susceptibility numerically, we note that the expectation value of the total (complex) electric dipole moment will be proportional to

$$\langle (\sigma_a + r\sigma_b) \rangle = e^{-i\omega_a t} (\rho_{ee,ge} + \rho_{eg,gg}) \\ + r e^{-i\omega_b t} (\rho_{ee,eg} + \rho_{ge,gg}) \quad (23)$$

For sufficiently small Ω_p , we expect that this quantity will be linear in Ω_p , and that the proportionality coefficient will be proportional to the probe susceptibility. We would also expect (23) to oscillate at the probe frequency ω_p , but, in fact, when the rotating-wave approximation of the previous section does not completely hold, we also

find components at other frequencies, due to the presence of the coupling field Ω_c ; hence, to estimate the component of the dipole moment that would contribute to absorption at the probe frequency, we take (23), multiply by $e^{i\omega_p t}$ and do a time average over a couple of inverse decay times (more precisely, from $t = 2/\Gamma$ to $t = 4/\Gamma$, to avoid the initial transients); then we divide this by Ω_p . The result may be taken to be an approximation to χ , in units of $|\chi_0|$ (see III.B, below, for the definition of $|\chi_0|$ adopted in this Section).

The above approach (which treats Ω_c and Ω_p as constants) should yield a reasonably accurate estimate of the absorption coefficient for sufficiently long pulses, i.e., longer than a few times $1/\Gamma$. For very short pulses, however, or when the two frequencies ω_p and ω_c are very close ($|\omega_c - \omega_p| \lesssim \Gamma$), this approach may not be very reliable, and the actual absorption of the probe would have to be calculated by solving coupled equations for the propagation of the fields in the medium. This is clearly beyond the scope of the present paper, although it is a question we intend to return to eventually, hopefully in conjunction with experimental results.

B. Identical dots

As explained in the previous Section, when the dots are identical one has an effective three level system, involving only the states $|ee\rangle, |+\rangle$ and $|gg\rangle$. Use of the decay equations (19) and (20), with $r = r' = 1$, shows that the population (diagonal density matrix element) of $|ee\rangle$ decays into $|+\rangle$ at a rate 4γ , while $|+\rangle$ decays into $|gg\rangle$ also at the rate 4γ (this is twice the rate used in Section II for the state vector amplitudes, as it should be; see Eq. (8)). The coherences (off-diagonal density matrix elements), on the other hand, have the decay rates $\dot{\rho}_{ee,+} = -(4\gamma + 2\Gamma)\rho_{ee,+}$; $\dot{\rho}_{ee,gg} = -(2\gamma + 8\Gamma)\rho_{ee,gg}$; and $\dot{\rho}_{+,gg} = -(2\gamma + 2\Gamma)\rho_{+,gg}$.

Figure 2 shows the dependence of the absorption coefficient (imaginary part of χ , where the negative sign indicates absorption) on the amplitude of the coupling field, Ω_c , when the probe and coupling field detunings, δ_p and δ_c of Eq. (14), are both taken to be zero, that is, $\omega_c = \omega_0 + d' - d$, $\omega_p = \omega_0 + d$. We have taken the dephasing rate Γ to be an order of magnitude larger than the radiative decay rate: $\gamma = 0.1\Gamma$. Since the overall decay rate of the coherence $\rho_{+,gg}$, which is the only relevant one in the limit $\Omega_c = 0$, is $2\gamma + 2\Gamma$, we have defined χ_0 as $\chi_0 = -i\mu^2/(\hbar\epsilon_0(\gamma + \Gamma))$ for the purpose of the scaling of the vertical axis.

From the analysis in Section II it follows that for equal dots the effects of d and d' can be combined into a single quantity, $2d - d'$, which is the difference between the resonant frequencies of the $|gg\rangle \rightarrow |+\rangle$ and $|+\rangle \rightarrow |gg\rangle$ transitions. Accordingly, the various curves in Fig. 2 are labeled by the value of this quantity. We see that when $|2d - d'| \gg \Gamma$, the simple analytical treatment in Section II gives qualitatively correct results: increasing the cou-

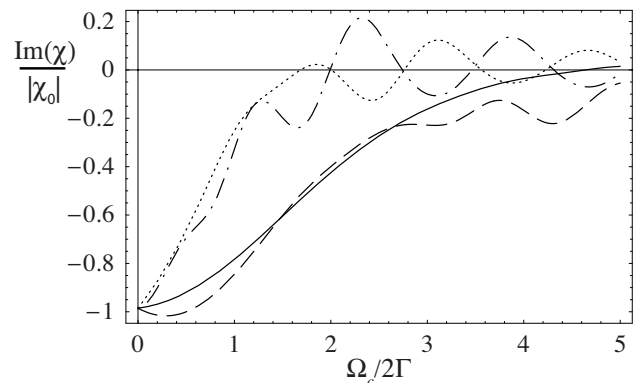


FIG. 2: Absorption rate of the probe as a function of the strength of the coupling beam, $\Omega_c/2\Gamma$, for $|2d - d'| = 20\Gamma$ (solid line), 10Γ (dashed), 5Γ (dotted), and 3Γ (dash-dotted).

pling beam reduces the absorption of the probe, and the relevant scale for Ω_c is of the order of a few times the largest decay rate in the problem.

On the other hand, when $|2d - d'|$ becomes comparable to the broadening Γ , the possibility of the coupling beam directly causing transitions from $|gg\rangle$ to $|+\rangle$ becomes non-negligible, especially for large values of Ω_c . The positive values of $\text{Im}(\chi)$ seen in the figures for some parameters suggest that it may be possible in this case to find parameter ranges where energy is transferred from the coupling beam to the probe. The narrower absorption dip observed in this regime is not necessarily bad news, since it suggests that switching may in fact be achieved with a smaller Ω_c , i.e., less power in the coupling beam.

Clearly, if d and d' have the same sign they will actually work against each other, in the sense of making the two transitions less distinguishable. Calculations such as those presented in Ref. 7 do indeed predict equal signs (both positive) for d and d' , but one may note that at least two experiments^{2,9} have reported what amounts to a negative d' , arising from an attractive interdot Coulomb interaction in the biexciton state $|ee\rangle$.

C. Non-identical dots

For this case one has a very large array of parameters, and some more or less arbitrary choices need to be made. As in the previous section, we assume dephasing is the main broadening mechanism and set $\gamma = 0.1\Gamma$. We also set $r = r' = 1.1$, i.e., we assume a 10% difference among the dots, both in the way they couple to the radiation field and in their dephasing rates.

Figure 3 shows several examples of the variation of the absorption with the coupling-field strength, where the fields' frequencies are assumed to be tuned to the $|0\rangle \rightarrow |2\rangle$ transition (for the probe field) and the $|2\rangle \rightarrow |3\rangle$

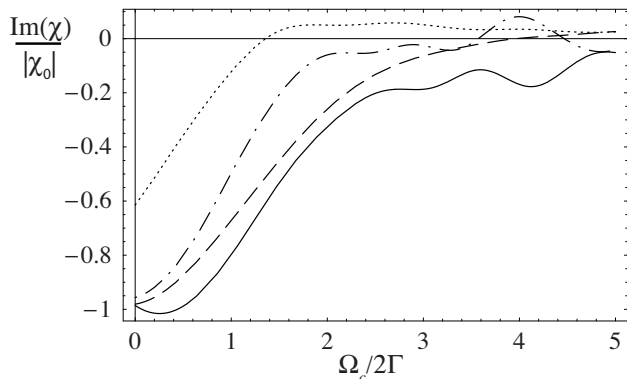


FIG. 3: Absorption rate of the probe as a function of the strength of the coupling beam, $\Omega_c/2\Gamma$, for unequal dots, and the probe and field frequencies given by Eq. (24). For all curves, $r = r' = 1.1$. Solid line: $\omega_{ab} = 0.5\Gamma$, $d = 5\Gamma$, $d' = 0$. Dashed line: $\omega_{ab} = 5\Gamma$, $d = 10\Gamma$, $d' = 0$. Dotted line: $\omega_{ab} = 5\Gamma$, $d = 0$, $d' = 10\Gamma$. Dash-dotted line: $\omega_{ab} = 5\Gamma$, $d = 5\Gamma$, $d' = 3\Gamma$.

transition (for the coupling field); that is, we have

$$\begin{aligned}\omega_p &= \frac{1}{2}(\omega_a + \omega_b) + \frac{1}{2}\sqrt{\omega_{ab}^2 + 4d^2} \\ \omega_c &= d' + \frac{1}{2}(\omega_a + \omega_b) - \frac{1}{2}\sqrt{\omega_{ab}^2 + 4d^2}\end{aligned}\quad (24)$$

The solid line in Figure 4 is for the case $\omega_{ab} < \Gamma$, in which the difference between the energies of the levels $|eg\rangle$ and $|ge\rangle$ is not easily resolvable, and hence the results do not differ very much from those of the previous section (compare to the dashed curve in Fig. 2).

The other three lines in Fig. 3 are for a large, resolvable splitting of the middle levels: $\omega_{ab} = 5\gamma$. The results of the numerical integration agree qualitatively with the claims in section II.B, that switching was also possible in this case, either by a sufficiently large d (dashed curve), or a sufficiently large d' (dotted line), or both (dash-dotted line). Again, one has to be careful, however, since it is essential to keep the two frequencies (24) sufficiently distinct, and, as discussed in the previous subsection, a positive d' may cause an accidental degeneracy; such a near-degeneracy accounts, in fact, for the somewhat unusual appearance of the dotted line in Fig. 3.

IV. DISCUSSION AND CONCLUSIONS

In conclusion, we see that pairs of quantum dots coupled by the dipole interaction, whether it is through the Förster mechanism or direct dipole-dipole interaction of excitons, can be used for optical switching under very general conditions, provided only that the rates associated with the interaction (d or d') be greater than about $10\Gamma = 5/T_2$. With $\hbar d \sim 1$ meV, the condition $d \geq 5/T_2$

becomes $T_2 \geq 3$ ps, which should be easily satisfied at sufficiently low temperatures. For instance, Borri et al.¹⁵ have observed dephasing times of about 10 ps for InGaAs dots at 100 K. We also note that the studies by Lovett et al.⁷ indicate that it is possible to increase d' by applying an external electric field, so it may not be unreasonable to assume couplings of the order of a few meV, which may allow for correspondingly larger decay rates, and higher operation temperatures.

As for the energies required to do the switching, it seems that Ω_c of the order of 3Γ is enough in most cases to reduce the linear absorption coefficient by a factor of about 2. In terms of the (real) electric field amplitude E_c we have $\Omega_c = \mu E_c/\hbar$, where again we can assume that the dipole moment matrix element μ is of the order of magnitude of ea , where a is the dot size. The MBE-grown dots on which we would like to perform this experiment¹⁷ are relatively large, about 6 nm in height and 20–30 nm in diameter. Assuming $a = 4$ nm in order to estimate μ , and $T_2 \simeq 10$ ps, we find that intensities of the order of $\epsilon_0 E_c^2 c/2 \simeq 100$ W/cm² may be required, which should be easily achievable with a short-pulse laser.

While more detailed calculations need to be made to fully ascertain how this system would behave in a transient, short-pulse regime, the results presented here suggest that, unlike quantum information applications, which require very precise characterization and tuning of all the experimental parameters, optical switching by pairs of coupled quantum dots should be a relatively robust effect. We hope to be able to demonstrate it experimentally in MBE-grown semiconductor quantum dots, and specifically in bilayer InAs/GaAs quantum dot structures, such as the ones described in Ref. 17.

Acknowledgments

This research has been supported by the ARO (Army Research Office) under grant #W911NF-05-1-0353.

APPENDIX A: DENSITY MATRIX EQUATIONS

We use an interaction picture in which the terms proportional to ω_a and ω_b in the Hamiltonian (15) have been removed, and define $\omega_{ab} = \omega_a - \omega_b$, and $\omega_{ij} = \omega_i - \omega_j$, where $i = p, c$ stands for the probe or coupling laser, and $j = a, b$. With the definitions

$$A = \frac{i}{2}(\Omega_p e^{i\omega_{pa}t} + \Omega_c e^{i\omega_{ca}t}) \quad (A1a)$$

$$B = \frac{ir}{2}(\Omega_p e^{i\omega_{pb}t} + \Omega_c e^{i\omega_{cb}t}) \quad (A1b)$$

the full density matrix equations that follow from Eqs. (18), (19) and (22) are

$$\dot{\rho}_{ee,ee} = -2\gamma(1+r^2)\rho_{ee,ee} + B\rho_{ee,eg} + A\rho_{ee,ge} + B^*\rho_{eg,ee} + A^*\rho_{ge,ee} \quad (\text{A2})$$

$$\begin{aligned} \dot{\rho}_{eg,eg} = & 2r^2\gamma\rho_{ee,ee} - 2\gamma\rho_{eg,eg} - (r\gamma - id)e^{-i\omega_{ab}t}\rho_{eg,ge} - (r\gamma + id)e^{i\omega_{ab}t}\rho_{ge,eg} - B\rho_{ee,eg} - B^*\rho_{eg,ee} \\ & + A\rho_{eg,gg} + A^*\rho_{gg,eg} \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \dot{\rho}_{ge,ge} = & 2\gamma\rho_{ee,ee} - 2r^2\gamma\rho_{ge,ge} - (r\gamma - id)e^{i\omega_{ab}t}\rho_{ge,eg} - (r\gamma + id)e^{-i\omega_{ab}t}\rho_{eg,ge} - A\rho_{ee,ge} + B^*\rho_{gg,ge} \\ & + B\rho_{ge,gg} - A^*\rho_{ge,ee} \end{aligned} \quad (\text{A4})$$

$$\dot{\rho}_{gg,gg} = 2\gamma\rho_{eg,eg} + 2r\gamma e^{-i\omega_{ab}t}\rho_{eg,ge} + 2r\gamma e^{i\omega_{ab}t}\rho_{ge,eg} + 2r^2\gamma\rho_{ge,ge} - B\rho_{ge,gg} - A^*\rho_{gg,eg} - B^*\rho_{gg,ge} - A\rho_{eg,gg} \quad (\text{A5})$$

$$\dot{\rho}_{ee,eg} = -(2\Gamma r'^2 + \gamma(2+r^2) + id')\rho_{ee,eg} - (r\gamma - id)e^{-i\omega_{ab}t}\rho_{ee,ge} - B^*\rho_{ee,ee} + A\rho_{ee,gg} + B^*\rho_{eg,eg} + A^*\rho_{ge,ge} \quad (\text{A6})$$

$$\dot{\rho}_{ee,ge} = -(2\Gamma + \gamma(1+2r^2) + id')\rho_{ee,ge} - (r\gamma - id)e^{i\omega_{ab}t}\rho_{ee,eg} - A^*\rho_{ee,ee} + B\rho_{ee,gg} + B^*\rho_{eg,ge} + A^*\rho_{ge,ge} \quad (\text{A7})$$

$$\dot{\rho}_{ee,gg} = -(2\Gamma(1+r')^2 + \gamma(1+r^2) + id')\rho_{ee,gg} + A^*\rho_{ge,gg} + B^*\rho_{eg,gg} - A^*\rho_{ee,eg} - B^*\rho_{ee,ge} \quad (\text{A8})$$

$$\begin{aligned} \dot{\rho}_{eg,ge} = & 2r\gamma e^{i\omega_{ab}t}\rho_{ee,ee} - (2\Gamma(1-r')^2 + \gamma(1+r^2))\rho_{eg,ge} - (r\gamma - id)e^{i\omega_{ab}t}\rho_{eg,eg} - (r\gamma + id)e^{i\omega_{ab}t}\rho_{ge,ge} \\ & + B\rho_{eg,gg} - B\rho_{ee,ge} + A^*\rho_{gg,ge} - A^*\rho_{ge,ee} \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} \dot{\rho}_{eg,gg} = & -(2\Gamma + \gamma)\rho_{eg,gg} + 2r\gamma e^{i\omega_{ab}t}\rho_{ee,eg} + 2r^2\gamma\rho_{ee,ge} - (r\gamma + id)e^{i\omega_{ab}t}\rho_{ge,gg} - B\rho_{ee,gg} - B^*\rho_{eg,ge} \\ & + A^*\rho_{gg,gg} - A\rho_{eg,eg} \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \dot{\rho}_{ge,gg} = & -(2\Gamma r'^2 + r^2\gamma)\rho_{ge,gg} + 2\gamma\rho_{ee,eg} + 2r\gamma e^{i\omega_{ab}t}\rho_{ee,ge} - (r\gamma + id)e^{-i\omega_{ab}t}\rho_{eg,gg} - A\rho_{ee,gg} - A^*\rho_{ge,ge} \\ & + B^*\rho_{gg,gg} - B^*\rho_{ge,ge} \end{aligned} \quad (\text{A11})$$

These equations can be modified to describe the case of totally uncorrelated dephasing (see the discussion in Section III.A) very simply, by expanding the binomials

$(1+r')^2$ (in (A.8)) and $(1-r')^2$ (in (A.9)), keeping the r'^2 term, and discarding the term linear in r' .

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