Homework Assignments

**Cartesian Tensors**

C1. Derive the transformation law for first-order Cartesian tensors: $A'_i = a_{ij} A_j$

C2. Derive the transformation law for first-order Cartesian tensors: $A_i = a_{ji} A'_j$

Using index notation, prove the following identities:

N1. $A \cdot (B \times C) = (A \times B) \cdot C$

N2. $\nabla (A \cdot B) = (B \cdot \nabla )A + (A \cdot \nabla )B + B \times (\nabla \times A) + A \times (\nabla \times B)$

N3. $(A \times B) \cdot (C \times D) + (B \times C) \cdot (A \times D) + (C \times A) \cdot (B \times D) = 0$

N4. $(A \times B) \cdot (B \times C) \times (C \times A) = (A \cdot B \times C)^2$

N5. $A \times (B \times C) + B \times (C \times A) + C \times (A \times B) = 0$

N6. $(A \times B) \cdot (C \times D) = (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)$

N7. $(A \times B) \times (C \times D) = B(A \cdot C \times D) - A(B \cdot C \times D) = C(A \cdot B \times D) - D(A \cdot B \times C)$

N8. $\nabla \cdot (\nabla \times A) = 0$

**General Tensors**

G1. Derive the transformation law for first-order covariant tensors: $A_i = \frac{\partial \bar{x}^j}{\partial x^i} A_j$

G2. Derive the transformation law for first-order covariant tensors: $\bar{A}_i = \frac{\partial x^j}{\partial \bar{x}^i} A_j$

G3. Derive the transformation law for first-order contravariant tensors: $A^i = \frac{\partial x^i}{\partial \bar{x}^j} \bar{A}^j$

G4. Derive the transformation law for first-order contravariant tensors: $\bar{A}^i = \frac{\partial \bar{x}^i}{\partial x^j} A^j$
Chapter 8 Matrices

M1. Determine a square root of the matrix $A$ as shown.

$$A = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$$

M2. Determine a square root of the matrix $B$ as shown.

$$B = \begin{bmatrix} -12 & 11.5 & -63 \\ -21 & 9.25 & -63 \\ 3 & -3.25 & 18 \end{bmatrix}$$

M3. Determine a square root of the matrix $C$ as shown.

$$C = \begin{bmatrix} -7 & 2.5 & 33 \\ 2.5 & 7.25 & -7.5 \\ -1 & 5 & 7 \end{bmatrix}$$

Chapter 7 Vectors

S1. Find the equation of the line which passes through the point $(5, 2, 3)$ and is parallel to the line of intersection of the plane containing the points $(1, 1, 1)$, $(1, 2, 4)$, and $(-1, 3, 5)$ and the plane containing the points $(2, 4, 2)$, $(3, -1, 2)$, and $(-2, -4, 3)$.

Ans. \[
\frac{x-5}{85} = \frac{y-2}{-33} = \frac{z-3}{-14}
\]
**S2.** Find the equation of the plane which contains the points (-1, -1, 1), (1, -1, 2), and (2, 1, -3).
Ans. $2x - 11y - 4z - 5 = 0$

**S3.** Find the shortest distance from the point (2, 4, 5) to the plane which contains the points (-1, -1, 1), (1, -1, 2), and (2, 1, -3).
Ans. $65/\sqrt{141}$ or 5.47

**S4.** Find the shortest distance between the line passing through the points $P(5, 0, 0)$ and $Q(4, 8, -4)$ and the line passing through the points $R(0, 0, 12)$ and $S(4, 7, 8)$.
Ans. $448/\sqrt{1937}$ or 10.18

**S5.** Solve Prob. S-4 for $P(5, 0, 0), Q(3, 2, 1), R(0, 0, 4)$ and $S(2, 6, 1)$.
Ans. $1/\sqrt{26}$ or 0.1961

**S6.** Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and has one half of its magnitude.

**S7.** Prove that the medians of a triangle trisect themselves.

**S8.** Find the shortest distance from the point (6, -4, 4) to the line passing through the points (2, 1, 2) and (3, -1, 4).
Ans. 3

**S9.** A force $\mathbf{F} = 6x \mathbf{j}$ N acts on a particle during its motion from $A_1(0, 0)$ to $A_2(1, 1)$ m along the path $C$ defined by $y = x$. Determine the work done by $\mathbf{F}$ on the particle.
Ans. 3 J

**S10.** Solve Prob. S-9 if the path $C$ is defined by $y = x(2 - x)$.
Ans. 2 J

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**Chapter 9 Vector Calculus**

- p. 496 8, 10
- p. 497 20, 22, 34
- p. 523 1, 7
- p. 524 23, 27
- p. 531 1, 3, 5, 25
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- p. 537 1
- p. 538 5, 7, 10
- p. 548 9, 13, 14
- p. 555 1, 24, 6