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Cut Saturation for *p*-cycle Design

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Abstract

Preconfigured protection cycles (p-cycles) provide recovery times for mesh networks that are near the recovery times of BLSR ring networks while providing lower spare-to-working capacity ratios. However, the efficiency of p-cycles defined by the spare-to-working capacity ratio is affected by the topology of the network. An efficient method for adding links to an existing network is required to take advantage of p-cycles. In this work, a heuristic called p-cycle cut saturation (PCUT) is proposed. It extends the well-known cut saturation heuristic to strategically add links to an existing network to lower the spare-to-working capacity ratios in networks using p-cycles. PCUT is applied to a 28-node network to add spans and lowers the spare-to-working capacity ratio by 20%.

1. Introduction

The increasing of number of business functions that rely on communication networks continues to increase significantly driving the demand for higher bandwidth services. Efficient protection of optical transport networks is important for providing reliable and available networks. Two well-known basic approaches for protecting optical networks are ring-based restoration [1], and mesh-based restoration methods such as span restoration [2], shared backup path protection [3], and path protection [4], [5], [6].

Today, most transport networks are based on rings. Rings are easy to manage and offer a fast way for protection switching. Unfortunately, they are capacity inefficient and the typical dedicated protection switched ring (DPRing) requires more than twice the working capacity for protection [7]. Mesh-based networks require much less spare capacity, but have the drawback of complicated protection mechanisms. The protection may be based on paths or on spans. A span is defined as connectivity between two adjacent nodes that contain one or more logical links. Sharing of the spare resources for protection often requires complicated signaling and therefore takes much longer than the simple switching required in ring networks.

Therefore, several new protection techniques for the transition of ring networks towards mesh-based networks have been proposed [8]. A very promising one, the concept of preconfigured protection cycles (*p*-cycles), has been introduced by Grover and Stamatelakis [9]. In *p*-cycles, the spare capacity for span protection is organized in cycles and shared among the on-cycle links and the straddling links of the cycle. The required percentage of required spare capacity, also known as redundancy, can be as low as 34% for *p*-cycles, while retaining the recovery speed of ring-based protection [10].

Grover and Stamatelakis [11] compare ring and mesh as two basic network restoration methods. The rings use a simple switching mechanism (low-cost add-drop multiplexers), which permits restoration times of approximately 50 ms, but they require at least 100% redundancy. In complete multi-ring network designs the overall redundancy or spare-to-working capacity ratio can be as high as 300%. Thus, rings are fast but not capacity-efficient. Mesh is more capacity-efficient because each unit of spare capacity is reusable in more ways across the network. Mesh restoration is based on the use of digital cross-connect systems. A span-restorable mesh networks may only require 50–70% redundancy, depending on network topology [12], [13].

Schupke, Gruber, and Autenrieth [14] investigate the deployment of *p*-cycles in WDM mesh networks with wavelength conversion or virtual wavelength path (VWP) and without wavelength conversion or wavelength path (WP). They apply *p*-cycle on a case

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study for a pan-European network. The results in VWP reach a spare-to-working capacity ratio of 36% for a constraint of *p*-cycles less than 5000 km, while in WP networks the spare-to-working capacity ratio is 71%. In addition, the *p*-cycle optimization performed better with a demand routing which tries to achieve balanced load on the links rather than using a shortest path routing algorithm. The *p*-cycles selection tends to choose cycles with higher number of nodes.

Doucette, He, Grover, and Yang [15] developed and tested an algorithmic approach for efficient enumeration of candidate *p*-cycles and capacitated *p*-cycle network design. The basic approach is to first identify a set of primary *p*-cycles, search for improvements on those cycles through various operations to create a final set of cycles of high individual and collective efficiency, and finally place one *p*-cycle at a time, iteratively, until all working capacity of the network is protected. The primary advantage of this algorithmic approach is that it entirely avoids the step of enumerating all cycles.

Grover and Doucette [10] introduced two pre-selection metrics of candidate *p*-cycles, topological score (TS) and apriori efficiency (AE), to reduce the complexity of solving optimal *p*-cycle design problems especially with joint capacitated problem (JCP). JCP jointly solves routing the working traffic and establishing *p*-cycles. JCP reduces the total required capacity by as much as 28% in span restoration, but only 10% in path restoration [13], [16].

Sack and Grover [17] show that in a homogeneous Hamiltonian network a Hamiltonian *p*-cycle is the most efficient overall solution, although interestingly it does not always correspond to the individually most efficient *p*-cycle that can be formed. They also introduce the concept of semi-homogenous networks, which can achieve the theoretical lower bound on span-restorable networks in terms of network redundancy.

This paper is organized as follows. In section 2, *p*-cycles are described. Section 3 describes wavelength division multiplexing. The spare capacity placement (SCP) and the joint capacity placement (JCP) formulations for assigning *p*-cycles are described in section 4. The proposed *p*-cycle cut saturation (PCUT) heuristic is described in section 5. PCUT is applied to the NSF network in section 6 and the results are discussed in section 7. Finally, conclusions about PCUT are presented in section 8.

2. *p*-cycles

Similar to bidirectional line switched ring (BLSR), the *p*-cycle concept is based on cycle structures. Spare capacity is reserved along the cycle. No signaling is needed in the network because of the ring-structure. The two nodes at the endpoints of the failure detour the traffic along these pre-configured protection capacities until the traffic reaches the target node. The reserved capacity is shared among multiple elements, providing restoration for one failure per cycle.

Unlike conventional ring concepts, *p*-cycles are formed in a mesh restorable network [11]. The traffic traversing an element can be divided into several *p*-cycles and may not belong to one ring as it is in BLSR. Because of the flexibility of the digital cross connects, the set of cycles which protect the network can be easily rearranged and do not have a structural association to their elements. Thus, the routing process can be performed freely on shortest paths. An optimal set of *p*-cycles that is able to protect the network is established in the remaining spare capacity of the network. The concept combines the flexibility of mesh networks with the fast restoration times of ring protection structures. Additionally, the cycle structure is accessible by more spans than just those lying on the link. Thus, the capacity efficiency of a mesh restorable network can be reached [18].

Fig. 1 shows an example of a *p*-cycle that is able to protect every link of the network. In the case of a single link failure, its traffic demand is detoured along previous reserved capacity units along the cycle.

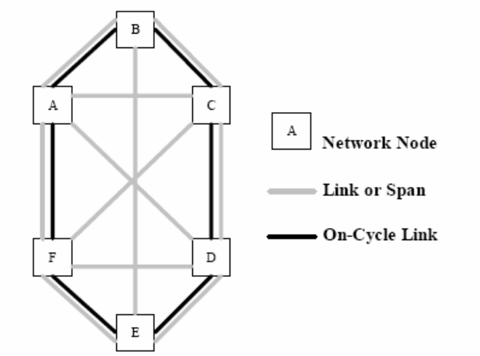

Figure 1. Example network topology

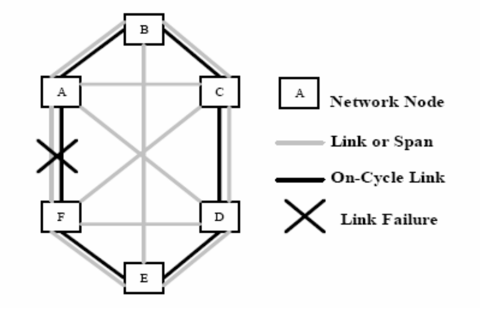

Figure 2. On-cycle span failure

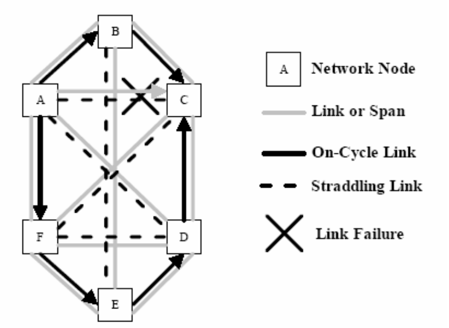

Figure 3. Straddling span failure

An example is given in Fig. 2 in which link A-F fails. When the adjacent nodes detect the signal loss of the link, their switches are changed accordingly. As a consequence nodes A and F will loop back the demand, which should be originally sent to the broken link, to the reserved capacities of the *p*-cycle. Because of the pre-configuration of the *p*-cycle, only two nodes adjacent to the link-failure must perform any real time actions.

Despite this similarity to today's well-known ring-concepts like BLSR and their optical versions optical path protection ring (OPPR) and optical shared protection ring (OSPR) [11], respectively, the same *p*-cycle is accessible to more links than for just those lying on the cycle [19], [18], [20]. Straddling links are links with end-nodes that lie on the cycle but they themselves are not part of the cycle. *P*-cycles are also able to protect these links. As seen in Fig. 3, demand originating at node A destined for node C can be detoured along the cycle until the demand reaches its destination.

Two restoration paths are available for straddling links. With this functionality, even a large number of demand capacity-units can be protected by a relative small *p*-cycle. The demand can be divided into two parts, each part taking its own direction along the cycle as seen in Fig. 3. Therefore, *p*-cycles are able to protect every link whose end nodes lay on the cycle.

Protecting straddling links during failure is fundamental to obtaining the efficiency of a mesh network using *p*-cycles. The reserved bandwidth on the *p*-cycle is shared with more links than in a ring. The *p*-cycle of Fig. 1 is able to protect 11 individual links. By protecting 6 on-cycle links and providing two restoration paths for 5 straddling links, it is more widely accessible than the same cycle in BLSR, which would be only able to protect 6 links.

3. WDM

This section presents the concept of *p*-cycles for wavelength division multiplexing (WDM) networks. The deployment of *p*-cycles and their ability to protect a network against a link failure is discussed. The main focus is the utilization of reserved spare wavelengths and their capacity efficiency compared to mesh restorable networks.

WDM restoration requires the discrete assignment of working as well as spare wavelengths. All wavelengths that may be used must be reserved in advance of any failure. In general, spare wavelengths do not carry demand traffic in a failure-free network (Although, it is possible to route low priority demand on spare wavelengths). Spare capacity is reserved to bypass demand wavelengths around a failure until other mechanisms react. In the case of a failure, the affected wavelengths are detoured to the protecting spare wavelengths. Spare capacity is shared among several links to minimize the number of reserved wavelengths. Due to this sharing, only one failure that is protected by the same *p*-cycle can be restored.

To protect the network, a set of cycles that minimizes the sum of needed spare wavelengths must be found. Thus, a main indicator to measure the efficiency of the *p*-cycle concept in WDM networks is the number of reserved spare wavelengths. If different networks are compared with each other, the ratio of the number of spare wavelengths to the number of demand wavelengths is a good metric. This is called the spare-to-working capacity ratio.

There are two types of WDM networks: virtual wavelength path (VWP) and wavelength path (WP). The nodes in VWP WDM networks perform full wavelength conversion, i.e., lightpaths can be switched to a fiber output if there is a free wavelength channel. The node can convert the incoming wavelength to another wavelength to use the output channel. The nodes in WP WDM networks do not have wavelength conversion [14]. In this study we focus on deployment of *p*-cycles in VWP WDM networks assuming the VWP nodes will be available for a reasonable price.

4. SCP and JCP

Sets:

S set of spans between mesh cross connection points.

P set of elemental distinct simple cycles.

D set of all demand pairs.

WR set of all candidate working routes for each demand pair *r*.

Parameters:

c_j cost of a link (working or spare) assigned to span *j*.

W_j number of working links placed on span *j*.

$x_{i,j}$ number of paths a single copy of *p*-cycle *i* provides for restoration of failure of span *j* (2 if straddling span, 1 if on-cycle span, 0 otherwise).

$p_{i,j}$ number of spare links required on span *j* to build a single copy of *p*-cycle *i* (1 if *p*-cycle *i* passes over span *j*, 0 otherwise).

d_r number of demand units between end-end pair *r*.

$\zeta_j^{r,q}$ Takes on value of 1 if the q^{th} working route for demand pair *r* uses span *j*, 0 otherwise.

Variables:

n_i number of copies of *p*-cycle *i* used.

s_j number of spare links placed on span *j*.

w_j number of working wavelengths placed on span *j*.

$wf^{\,r,q}$ working capacity required by q^{th} working route for demand between node pair *r*.

4.1. Spare Capacity Placement (SCP) in VWP WDM networks

In spare capacity placement (SCP) the demand is first routed using shortest path, and then the optimal capacity algorithm is applied on the spare links to minimize the spare capacity cost as in (1) [9].

$$\text{Objective function: } Minimize \quad \sum_{j=1}^{|S|} c_j \times s_j \qquad (1)$$

Subject to:

$$s_j = \sum_{i=1}^{|P|} p_{i,j} \times n_i \qquad \forall j \in S \qquad (2)$$

$$W_j \le \sum_{i=1}^{|P|} x_{i,j} \times n_i \qquad \forall j \in S \qquad (3)$$

$$n_i \in \{0,1,2,........\} \qquad (4)$$

4.2. Joint Capacity Placement (JCP) in VWP WDM networks

In joint capacity placement (JCP) the working routes are optimized at the same time as restoration routes and spare capacity placement to minimize the total capacity as in (5) [21].

Objective function:

$$Minimize \quad \sum_{j=1}^{|S|} c_j \times w_j + \sum_{j=1}^{|S|} c_j \times s_j \qquad (5)$$

Subject to:

$$\sum_{q=1}^{|WR|} wf^{\,r,q} = d_r \qquad \forall q \in D \qquad (6)$$

$$w_j = \sum_{r=1}^{|D|} \sum_{q=1}^{|WR|} wf^{\,r,q} * \zeta_j^{r,q} \qquad \forall j \in S \qquad (7)$$

$$s_j = \sum_{i=1}^{|P|} p_{i,j} \times n_i \qquad \forall j \in S \qquad (8)$$

$$w_j \le \sum_{i=1}^{|P|} x_{i,j} \times n_i \qquad \forall j \in S \qquad (9)$$

$$n_i \in \{0,1,2,........\} \qquad (10)$$

Constraints (2) and (8) determine the protection capacity allocation, constraints (3) and (9) ensure the working capacity to be protected, and (4) and (10) ensure integer *p*-cycle units. Constraint (6) ensures all demands are routed, and constraint (7) ensures the working capacity on span *j* can accommodate all working flows simultaneously routed over it by all demand pairs.

5. *p*-cycle Cut Saturation

The cut saturation heuristic is one of the popular methods for designing communication networks [22]-[24] and designing optical networks with low cost and high utilization [25]. The idea in the cut saturation method is to identify a partition or cut of the nodes into two sets of networks N1 and N2 such that the links joining N1 and N2 are highly utilized. Links of high utilization are temporarily removed until the network separates into two components, N1 and N2. A link is then added that connects a node in N1 with a node in N2, while a link of low utilization is deleted.

The cut saturation method is a heuristic that identifies two groups of nodes that have higher traffic between them. Therefore, the only way to significantly distribute the traffic more uniformly is to connect a link between these two groups of nodes.

In this work, an extension to the existing cut saturation heuristic algorithm called *p*-cycle cut saturation (PCUT) was developed to reduce the spare-to-working ratio in an existing network that uses *p*-cycles by adding spans strategically to the network. PCUT is shown in Fig. 4. First, PCUT starts with initial network topology. Traffic requirements between all node pairs must be specified. The next step in PCUT is to route the demand using a shortest path routing algorithm. Then the spans are sorted from the highest utilization to the lowest utilization. The most saturated span is temporarily removed until the network is separated in two disconnected components N1 and N2. Then a new span is added that connects a node with the lowest node degree in N1 and a node with the lowest node degree in N2. The spare-to-working ratio is calculated by applying either SCP or JCP. If the spare-to-working capacity ratio is the desired value, then stop. Otherwise, replace the network topology with a network that has the new span and go back to routing the demand in the new network.

```
Load topology and traffic demand.
do {
    Route traffic with JCP or with shortest path algorithm if using SCP.
    Sort spans by load in descending order.
    Remove spans one at a time until the network separates into two disconnected components N1 and N2.
    Add a new span between one of the lowest degree nodes in N1 and one of the lowest degree nodes in N2.
    Replace deleted spans.
    Find p-cycles with SCP or JCP.
    Calculate spare-to-working capacity ratio.
} while (spare-to-working capacity ratio is greater than a threshold)
```

Figure 4. PCUT heuristic

6. PCUT to NSF Network

PCUT was applied to the National Science Foundation network (NSF) that has 28 nodes and 45 spans. The NSF network is a benchmark network for network design algorithms and is shown in Fig. 5. The number next to each span is the working wavelengths needed in each span after routing the randomly generated demand using the shortest path algorithm. The number of required wavelengths between each source and destination was generated randomly with a uniform distribution in the range from 1 to 15. Then PCUT adds spans strategically to the network to reduce the spare-to-working capacity ratios.

Each time a span is added to the network, the spare-to-working capacity ratio is evaluated and compared it to the original value, until a significant reduction in the spare-to-working capacity ratio is achieved. The results are shown in Table 1.

7. Results

The results of adding four additional spans to the network using PCUT are shown in Table 1. The average node degree $\bar{d}$, spare-to-working capacity ratio (S/W) for SCP, S/W for JCP, and a lower bound for a span-restorable network, $1/(\bar{d}-1)$ [9] are shown. PCUT decreases the S/W by 20% for SCP and by 20% for JCP after adding four spans to the network. Two additional spans are added for a total of six additions spans and the results are shown in the last row of Table 1. Adding the two additional spans does not significantly reduce the S/W. On this and other networks, when the average node degree is larger than approximately 3.5 then additional spans do not significantly reduce the S/W.

8. Conclusions

PCUT is a good strategy for adding spans to a network that uses *p*-cycles. It increases the number of straddling links in the network to take advantage of *p*-cycles. The advantage of PCUT is its application to larger networks where the computation requirements are too large to calculate every combination of additions. In the example 28-node network, PCUT decreased the spare-to-working capacity ratio by 20% by strategically adding four spans.

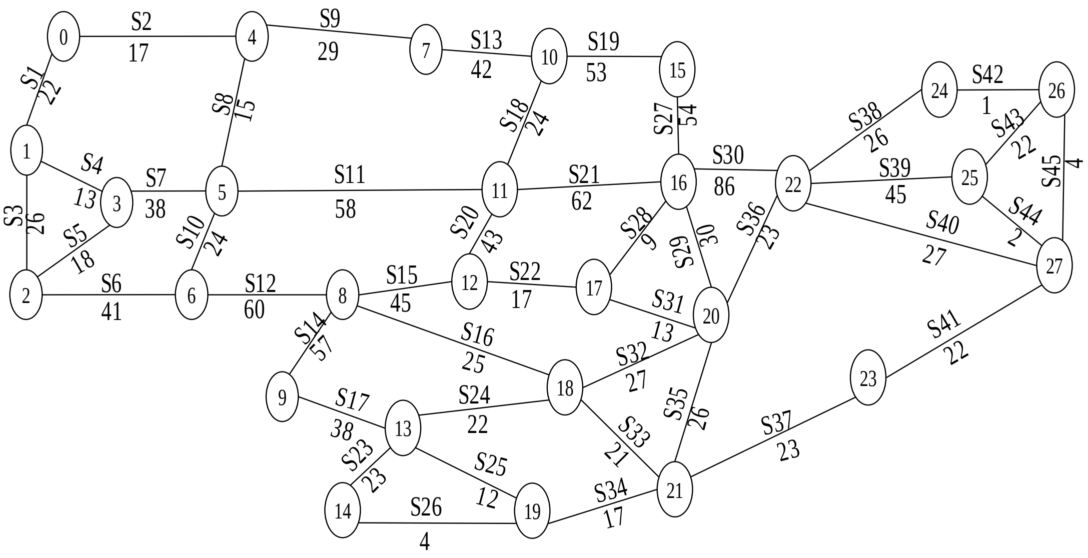

Figure 5. NSF Network

Table 1 Spare-to-working capacity ratio (S/W) for 1, 2, 3, 4, and 6 additional spans added by PCUT

The location of new spans	**Avg Node Degree**	**S/W SCP (%)**	**S/W JCP (%)**	**S/W Bound (%)**
Original network	3.21	81.78	60.08	45.25
N_0 to N_9	3.28	76.24	52.11	43.86
N_0 to N_9, N_7 to N_{14}	3.36	72.08	51.11	42.37
N_0 to N_9 , N_7 to N_{14}, N_{15} to N_{24}	3.43	68.84	49.64	41.15
N_0 to N_9, N_7 to N_{14}, N_{15} to N_{24}, N_{10} to N_{23}	3.5	65.65	47.96	40.00
N_0 to N_9, N_7 to N_{14}, N_{15} to N_{24}, N_{10} to N_{23}, N_0 to N_{14}, N_{15} to N_{23}	3.64	64.55	47.05	37.88

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