Leveraging Revealed Preference Information by Sequentially Pricing Multiple Products

John Aloysius  
Information Systems Dept.  
University of Arkansas

Cary Deck  
Economics Dept.  
University of Arkansas

Amy Farmer  
Economics Dept.  
University of Arkansas

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Technological advances enable sellers to price discriminate based upon a customer’s revealed purchasing intentions. E-tailers can track items in “shopping carts” and RFID tags enable retailers to do the same in bricks and mortar stores. In order to leverage this information, it is important to understand how this new visibility impacts pricing and market outcomes. We examine the theoretical implications of sequential pricing of multiple products that are independently-valued, are positively or negatively correlated, or have super-additive or sub-additive values. The results indicate that sequential pricing can increase profits relative to simultaneous components pricing for substitute goods due to a reduction of intraseller competition. When sellers can condition the second good’s price on the buyer’s decision to purchase the first good, sequential pricing can also increase profits relative to mixed bundling when customer’s values for the goods are highly positively correlated.

Key Words: Sequential Pricing, Price Discrimination, Bundling, Market Experiments

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1. Introduction

Imagine walking into a large department store and searching for a new outfit. The buyer observes a variety of shirts with posted prices, selects one, and then moves on to an area containing pants. The price of each pair of pants was set before the buyer selected a shirt and therefore the seller was dealing with a simultaneous pricing problem. One way sellers have attempted to exploit information on the underlying distribution of preferences among goods is by selling a collection of items in a bundle; however the prices are still set ex-ante. For example, the department store could sell an outfit rather than pants and shirts. The term pure bundling refers to offering only the bundle, whereas mixed bundling refers to offering both the individual items and the bundle simultaneously. Only offering the items separately is termed pure components pricing.

Now imagine that the seller is able to identify which shirt the customer selected (or even simply picked up for a moment) before setting the prices of the pants. The shirt selection reveals information about the buyer’s tastes and preferences, thereby enabling the seller to better estimate the buyer’s willingness to purchase any specific pair of pants. The seller could then effectively raise the price for items that are more likely to be purchased, perhaps by offering a smaller coupon for coordinating pants than for other pairs. Recent advances in technology have enabled exactly this type of monitoring of shoppers, so that pricing decisions for multiple products can be sequential rather than simultaneous.

Consider the situation where two myopic people, High (H) and Low (L), each value two products A and B. Suppose, $V_A^H = V_B^H = V$ and $V_A^L = V_B^L = v$ where $V > 2v > 0$ and the value of consuming the bundle containing both products is $V_A^i + V_B^i$ for $i = H, L$. For simplicity, assume that the marginal cost of each product is 0. Under pure components, the maximum attainable profit is $2V$, reaped by selling both products at a price of $V$ to the high valued buyer, H. The seller could obtain $4v$ from selling both items to both buyers at $v$, but by assumption $4v < 2V$. Under pure bundling, the maximum profit is again $2V$ and generated by selling the bundle to person H for $2V$. Even with mixed bundling the firm can only earn $2V$, all from person H, who will again purchase both goods. The inability of mixed bundling to increase profits is due to the fact that no discount can be offered for the bundle that would result in L buying the bundle while H bought the items separately. However, under sequential pricing with discrimination, the seller can obtain a profit of $2V + v$. Assuming that the decision to purchase A is made first, the seller can
set the price of good $A$ at $V$. $H$ will purchase $A$ and $L$ will not. The seller can then set the good $B$ price at $V$ for those who purchased $A$ and set the good $B$ price at $v$ for those who did not. In this case both $H$ and $L$ would buy good $B$ but at different prices.

The preceding illustration motivates the current research – what is the optimal pricing strategy for sellers who can monitor a customer’s initial purchase decision? The problem of conditional sequential pricing is one of first being able to identify the order in which prices are observed and also being able to identify the customer’s action and then exploit this information. The example also illustrates two key aspects of the seller’s problem: the degree to which buyers are forward looking and connect the two pricing decisions and the ability for products to be presented sequentially. There is evidence that humans are boundedly rational and thus some researchers question the appropriateness of modeling buyers as being fully rational (Ellison 2006). In particular, decision makers typically frame problems narrowly rather than broadly, deciding about local options without considering all alternatives (March 1994). People are content to find a set of sufficient conditions to solve a problem rather than the most efficient set of conditions. March (1994) also lists problem decomposition as a coping strategy: people attempt to decompose large problems (e.g., how to make two sequential purchase decisions) into their component parts. Such problem decomposition in the context of retail buying decisions may result in myopic decision behavior, since buyers focus on a product purchase without considering strategies that will impact future prices of other goods. Retailers already use technologies that also enable sequential pricing to make product recommendations, which implies that buyers are often unfamiliar with other items a seller has to offer. Assuming that buyers have formed expectations of prices for these yet to be recommended items when considering the initial item seems unrealistic. While the example above deals with two goods, many retailers carry a large number of SKUs and expecting a buyer to understand how each possible combination of purchases could affect future promotions also seems unrealistic. Therefore, our focus is primarily on the case of myopic buyers for this initial study.

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1 For the sake of this paper it does not matter if the presentation order is exogenous or endogenous as long as the seller can identify that an order exists either by tracking the buyer or by construction of the shopping experience. In bricks and mortar retail, the flow of people through the store is carefully arranged. For example, grocery stores generally have shoppers enter through the fresh produce and leave impulse items for the checkout stand. In online markets shoppers have to actively negotiate websites to observe prices and the seller can track this history. The optimal order in which to present goods in the face of individually tailored marketing promotions is an open empirical question.
Identification of which items a buyer has observed is straightforward in online markets where buyers place items in electronic shopping carts and cookies allow a shopper to be monitored. For example, when purchasing airline tickets through an online service, shoppers are often shown ads for hotels and attractions near their destination. It is easy to display a different set of ads and promotional offers to those who have booked a Saturday night stay (or purchased a child’s ticket). Can the same level of monitoring occur in physical stores? The answer is yes, due to ubiquitous computing technologies (Acquisti 2006). Currently, RFID (radio frequency identification) technology is being used to monitor which products buyers in bricks and mortar stores have in their physical shopping carts. This technology is being employed primarily for theft detection, but other applications are being explored by industry and academia (Cromhout et al. 2008). Retailers, such as the Dillard’s department store among others, have introduced item level tagging in pilot stores and are planning expansion of the program. Sam’s Club, Walmart’s retail warehouse club division, expanding on its previous pallet level tagging mandate, recently introduced an item level tagging mandate for its suppliers, requiring that they tag all items shipped to 22 distribution centers by 2010 (Weier 2008). The chain is poised to unveil a new RFID enabled customer checkout system that will considerably reduce transactions costs and improve inventory control. In physical stores current practices are such that all buyers observe the same quoted prices; however, the use of new smart shopping carts can transfer the price display from the shelf to the shopper’s cart allowing each shopper to receive a unique price or coupon. Alternatively, individualized coupons can be sent to each shopper’s phone. According to a December 17, 2008 article in the New York Times by Bob Tedeschi, this technology is already being employed by companies such as Cellfire and 8Coupons to offer coupons from retailers such as Sears and Kroger’s.

Sellers also need to know how the buyer’s value for an item is related to the buyer’s value for other items. For this purpose, sellers have access to vast databases that can be mined to determine underlying relationships in buyer values across goods (Ulph and Vulkan 2003). Sellers routinely record the contents of every shopping basket sold. If purchases are made with credit or debit cards or some other form of identification such as a frequent buyer cards, a customer’s shopping history within and across retailers can be compiled. Techniques like collaborative filtering and content filtering enable websites such as Amazon.com to provide recommendations to specific customers for other products

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2 United States Patent 5729697 is for just such a device. The carts are also touted as way to monitor the health content of a shopper’s purchases. Even without such technology, buyers may still pay different prices depending on the coupons they have, some of which could have been mailed specifically to them.
based upon the information in such databases. Ansari et al. (2000) point out that among other sources, a customer’s preferences or choices is information that can be used to make recommendations to customers. Knowing that one person is more likely then someone else to enjoy an item also suggests that the person is willing to pay more.  

To engage in any form of price discrimination, a seller must be able to identify their customers. Until recently however, such identification has been costly (Vulkan 2003). Moorthy (1984) points out, there are advantages of consumer self selection as the basis for market segmentation and price discrimination. Ulph and Vulkan (2003) refer to the ability to better tailor prices and thereby extract greater surplus caused by the wave of personalization in e-commerce as the enhanced surplus extraction effect. As previously described, technology enabled real-time monitoring of customers provides retailers with the ability to identify customers based upon revealed preferences and as a result, to better optimize pricing decisions taking advantage of both customer self-selection and enhanced surplus extraction.

This research considers two ways in which a buyer’s value for two products might be related – the degree of complementarity and the degree of correlation between the valuations of the two goods. Two products which have greater utility to a customer when they are consumed together are complements; the values of the two items are superadditive in that the bundle is worth more than the sum of its parts. Similarly two products are substitutes if the value of consuming both is less than the sum of values from consuming the single items; that is the values are subadditive. Notice that the complements/substitutes relationship is distinct from the correlation between the values of the two goods. Two books on a related topic can be substitutes or complements depending on the overlap in their content, but people who dislike the topic will likely have a low value for both, while people who like the topic will likely have a high value for both. The theoretical results presented in this paper suggest that sequential pricing with discrimination outperforms mixed bundling when the goods have

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3 Conditional sequential pricing may be classified as a form of third degree price discrimination (Pigou 2006). Ayres (2007) gives many examples of firms that currently engage in such practices, ranging from the most visible examples of supermarkets to those less visible such as Harrah’s casinos. Harrah’s records real-time data on players winning or losing, and in combination with demographic information, uses this information to offer complementary promotional benefits to players who lose more than a critical threshold amount. In this way they avert these players leaving with a negative experience from their visit to the casino.
highly positively correlated values. Sequential pricing even without discrimination is more profitable than simultaneous pricing of pure components when the goods are weak substitutes. The rest of this paper is organized as follows. Section 2 reviews research on pricing relevant to the current research framework. Section 3 outlines an analytical framework that models sequential pricing decisions faced by a monopolist. In Section 4 we summarize the results and offer comments.

2. Literature Review

Research on sequential pricing and exploiting the underlying relationship among the goods is surprisingly sparse given the rapid proliferation of technologies that enable retailers to gather information on likely purchases that could be used to set prices on those candidate goods in order to maximize expected profit. Mulhern and Leone (1991) review multi-product pricing and develop a framework for retail pricing and promotion policies. Using empirical data, they estimate the influence of regular and promotional prices on sales of substitute and complementary goods, and thus demonstrate the effectiveness of price promotions as a means of exploiting interdependencies in demand among retail products.

Instead, the literature on selling multiple products has been primarily focused on bundling by monopolists. Bundling has been shown to be an effective price discrimination tool even when the consumer’s willingness to pay for each good is independent of the value of the other good and the value of the bundle is the sum of the values of the components (Adams and Yellen 1976). Customers with a high degree of asymmetry in product valuations will buy an individual product that they favor, while customers with more symmetric valuations will buy the bundle. Venkatesh and Kamakura (2003) present an analytical model of contingent valuations and find that the degree of complementarity or substitutability in conjunction with marginal cost levels determines whether products should be sold as pure components, pure bundles, or mixed bundles. They also find that typically, complements and

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4 There are a few studies of price bundling in competitive markets. McAfee, McMillan, and Whinston (1989) extend their monopoly results to a duopoly and show that independent pricing can never be a Nash equilibrium when the reservation prices for the single goods are independent. Chen (1997) analyzes a situation in which firms compete in a duopoly for a single product and the firms also produce other products under conditions of perfect competition. Bundling as a product differentiation device proves to be an equilibrium strategy for one or both of the firms. Aloysius, Deck and Farmer (2009) report behavioral experiments where firms engage in bundling while competing for informed customers and maintaining monopoly power over uninformed customers.

5 Schmalensee (1984) finds similar results in a model with continuous (bivariate normal) valuations. McAfee, McMillan and Whinston (1989) provide conditions under which such bundle pricing is optimal. Hanson and Martin (1990) show how to compute optimal bundle prices using a mixed integer linear program.
substitutes should be priced higher than independently valued products. Nettesine, Savin, and Xiao (2006) present a stochastic dynamic program for analyzing the selection of complementary products.

There are numerous studies of dynamic pricing of goods, although these are confined to single goods and do not consider cross category effects on other goods. Cope (2006) presents dynamic strategies for maximizing revenue in internet retail by actively learning customers’ demand responses to price. Zhang and Krishnamurthi (2004) provide a decision support system of micro-level promotions in an internet shopping environment, that provides recommendations as to when, how much, and to whom to give price promotions. The system derives the optimal price promotion for each household, on each shopping trip by taking into account the time-varying pattern of purchase behavior and the impact of the promotion on future purchases.

There are several examples of sellers using customer behavior to infer preferences, and using that information either to drive revenues or for customer relationship management. Montgomery et al. (2004) show how clickstream data about the sequence of pages or path navigated by web buyers can be used to infer users’ goals and future path. There is a literature on behavior-based price discrimination (for a survey see Fudenberg and Villas-Boas 2006) in which firms use information about consumers’ previous purchases to offer different prices and/or products to consumers with different purchase histories. Empirical data shows that even the information contained in observing one historic purchase occasion by a customer boosts net target couponing revenue by 50% (Rossi et al. 1996). Automotive retailers can use information on which online infomediary customers use in order to market segment and price discriminate (Viswanathan et al 2007).

Better ability to predict preferences has been shown to potentially reduce price competition (Chen et al. 2001). Acquisti and Varian (2005) show analytically that it is optimal to price so as to distinguish between high-value and low-value customers. There is empirical evidence that competing firms have been able to price discriminate profitably by charging different prices across consumer segments (Basenko et al 2003). Moon and Russell (2008) develop a product recommendation model based on the principle that customer preference similarity stemming from prior purchase behavior is a key element in predicting current purchase. These studies exploit customer revealed preference for a good in order to set prices for future purchases of that good. The current research extends the issues explored in previous research to study how buyer revealed preferences inferred from initial purchase decisions of one good, can be used to set optimal prices for purchases of other goods. One advantage of this new mode of target marketing is that information on revealed preference can be used in the same
online or in-store visit. Furthermore, it is not dependent on identifying customers in order to access their buying history.

3. Sequential Pricing

3.1 Problem Description

We begin with the simple assumption of a seller facing a pricing decision in two sequentially ordered markets. The products may be substitutes, complements, or neither. However, it is assumed that the consumer’s value for one item is distributed independently of the other. In other words, the value of the product purchased second may depend on whether the initial good was purchased, but the values of the two goods separately are not correlated. Of course, the optimal pricing strategy depends upon whether the seller can use information regarding the consumer’s decision in the market for the first good when setting the price for the second good. We first consider the case in which the monopolist cannot use such information, perhaps due to technological constraints, fear of consumer backlash or government action. This is followed with an analysis of the case in which the monopolist is able to price discriminate in the market for the second good.

Before considering these two cases, it is useful to note that sequential pricing in the absence of price discrimination is substantively different from simultaneous pricing. The sequential pricing problem is one in which the seller recognizes the impact of the price of good A on the purchase of good B. By recognizing the behavioral response at the second stage (good B decision) to the outcome in the first stage (good A decision), the seller will consider those results in expectation when pricing good A. Consumer behavior differs in that the sequential problem does not provide the consumer with full information when making a choice. Rather, the consumer in this model is assumed to be myopic, choosing to purchase A solely on its price relative to value and then a choice regarding B will occur subsequently. Given that consumer behavior is entirely different due to the timing effects, the pricing strategy also is entirely different. As such, it is important to fully model the case of sequential pricing absent price discrimination.

We begin with the general market set-up and then consider the each case in turn. Assume a market exists for two products denoted A and B, and a consumer has a choice first to buy A followed by

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6 The buyer is assumed to only consider purchases from this seller effectively making the seller a monopolist in this market. A lack of comparison shopping could be due to actual market monopolization, high search or switching costs, or a preference for the particular seller. The order of the markets need not be controlled so long as the seller can identify the order before determining what price the buyer will observe for the second item.
a choice to buy B in sequence. Let the consumer’s value for A be distributed \( V_A \sim f_A(V_A) \). Then the consumer will buy A iff \( P_A \leq V_A \). Similarly, the consumer’s independent value for B follows the distribution \( V_B \sim f_B(V_B) \). Following Venkatesh and Kamakura (2003), a consumer’s joint value from purchasing both A and B is denoted \( V_{AB} = (1 + \theta)(V_A + V_B) \), where \( \theta \) represents complementarity if \( \theta > 0 \), and substitutability if \( \theta < 0 \). A consumer who chooses not to purchase A will buy B iff \( P_B \leq V_B \). However, if A was purchased, then the joint value becomes relevant and the consumer will buy B iff \( P_B < (1 + \theta)(V_A + V_B) - V_A \) which can be rewritten as \( V_B > \frac{P_B - \theta V_A}{1 + \theta} \). A consumer will purchase at most one unit of each item, and the monopolist produces each item at constant marginal costs of \( C_A \) and \( C_B \) respectively.

Given this framework, now consider a seller’s price setting problem.

**Case 1: Pricing without Price Discrimination**

When the seller sets \( P_B \), it is as if there is no information concerning the decision to buy A. Rather, the seller will know the probability that A will have been purchased, conditional on the price of A and the distribution of preferences. Similarly, the seller sets the price of A knowing the probability A will be bought and therefore, how that probability will affect the subsequent purchase of B. Let us consider these stages in reverse. In other words, conditional on a price of A and the corresponding probability that A was purchased, how then should the seller price B? And then, given that optimal response to a price of A, how should the seller price A in the first place?

**Stage 2: Price of B**

The seller will maximize expected profit with respect to the price of B conditional on a price of A and the distributions over preferences. Profits are \( P_B - C_B \) if a sale is made and 0 otherwise. The seller maximizes equation (1).

\[
\max_{P_B} E\Pi(P_B|P_A) = (P_B - C_B) \int_{P_B}^\infty \int_0^{P_A} f_A(V_A) f_B(V_B) dV_A dV_B + (P_B - C_B) \int_{P_A}^\infty \int_{P_B - \theta V_A}^{\infty} f_A(V_A) f_B(V_B) dV_B dV_A
\]

(1)

The first term is profit from those who buy B but not A and the second term is profit from those who buy both B and A. Differentiating (1) with respect to \( P_B \) to determine the first order condition and solving for \( P_B^* = f(P_A) \) gives an optimal response function based upon the choice in the first market.
Stage 1: Price of A

Given $P_B^*$, the seller must choose the optimal $P_A$ for stage 1 by maximizing expected profit for the sum of both stages, recognizing that the choice of $P_B$ depends on $P_A$. In stage 1, the seller maximizes equation (2).

$$\max_{P_A} E \Pi(P_A) = (P_A - C_A) \int_{P_A}^{\infty} \int_0^a \frac{p_B^{\theta \gamma A}}{1+\theta} f_A(V_A) f_B(V_B) dV_B dV_A$$

$$+ (P_B^* - C_B) \int_{P_B^*}^{\infty} \int_0^a f_A(V_A) f_B(V_B) dV_B$$

$$+ (P_A - C_A + P_B^* - C_B) \int_{P_A}^{\infty} \int_0^a \frac{p_B^{\theta \gamma A}}{1+\theta} f_A(V_A) f_B(V_B) dV_B dV_A$$

(2)

where the first term is profit from those who buy A only, the second term is profit from those who buy B only, and the third term is the profit from those who buy both. The general solution is derived by finding the first order condition of (2) with respect to $P_A$, solving this first order condition for $P_A^*$ and then calculating $P_B^*$.

This exercise is intractable in general, so we now consider the case of a uniform distribution for consumer preferences. Specifically, let $f_A(V_A) \sim U[0,100]$ and $f_B(V_B) \sim U[0,100]$. In this case (1) can be rewritten as

$$\max_{P_B} E \Pi(P_B | P_A) = (P_B - C_B) \int_{P_B}^{100} \int_0^a \frac{1}{100^2} dV_B dV_A + (P_B - C_B) \int_{P_B}^{100} \int_0^a \frac{1}{100^2} dV_B dV_A$$

(1*)

After integrating and simplifying (1*) the problem becomes

$$\max_{P_B} E \Pi(P_B | P_A) = \frac{(P_B - C_B)}{100^2} [100^2 - P_A P_B + \frac{P_B}{1+\theta} (P_A - 100) + \frac{\theta}{1+\theta} \left(\frac{100^2 - P_B^*}{2}\right)]$$

(1’*)

Differentiating (1’*) with respect to $P_B$ and simplifying the first order condition yields

$$P_B^* = \frac{-C_B (P_A - 100) - 100^2 (2+\theta) + \theta P_A^2}{2(P_A \theta + 100)}$$

(3)

Performing a similar exercise using uniform distribution, (2) can be rewritten as

$$\max_{P_A} E \Pi(P_A) = (P_A - C_A) \int_{P_A}^{100} \int_0^a \frac{p_B^{\theta \gamma A}}{1+\theta} \frac{1}{100^2} dV_B dV_A$$

$$+ (P_B^* - C_B) \int_{P_B^*}^{100} \int_0^a \frac{1}{100^2} dV_B dV_A + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{100} \int_0^a \frac{p_B^{\theta \gamma A}}{1+\theta} dV_B dV_A$$

(2’)*
Simplifying (2`) yields (2’)

$$\max_{P_A} E\Pi(P_A) = \frac{(P_A - C_A)}{100} (100 - P_A) + \frac{(P_B - C_B)}{100^2} \left[ 100^2 - P_A P_B^* + \frac{P_A P_B^* - 100PA}{1 + \theta} + \frac{\theta(100 - P_A)}{2(1 + \theta)} \right] \tag{2’}$$

where $P_B^*$ is found in equation (3). Solving the first order condition for (2’) we need to differentiate with respect to $P_A$ given that $P_B$ is a function of $P_A$. The first order condition is the following fourth order polynomial in $P_A$:

$$0 = (3P_A^4 \theta^3 + 8P_A^3 \theta^2 [-350 + (-400 + C_B) \theta] + 4P_A^2 \theta [-160000 + 400 (-325 + C_A + C_B) \theta + (25000 + 400C_A + C_B^2) \theta^2] + 800P_A [-40000 + 100 (-100 + 4C_A + 4C_B) \theta + (25000 + 400C_A + C_B^2) \theta^2] + 20000 [8000 + (6000 + C_B^2) \theta - 6000 \theta^2 - 45000^3 + 80C_A (1 + \theta)] [160000 (1 + \theta)(100 + P_A \theta)^2]^{-1}.$$  

Although quite cumbersome, real solutions can be found using Mathematica. Panel A in Figure 1 shows profit as a function of $P_A$ for values of $\theta$ varying from -0.5 (lower dark curve) to 1 (upper light gray curve) assuming that $P_B$ satisfies (3) and that $C_A = C_B = 0$. It is clear from this figure that profits increase in theta and that the optimal price of good A decreases with theta. Panel B of Figure 1 is similar, but it assumes that $\theta = 0$ and considers changes in $C_A$ and $C_B$. Black curves indicate that $C_A = 0$ while gray lines indicate that $C_A = 20$ and solid curves indicate that $C_B = 0$ while dashed lines indicate that $C_B = 20$. From this figure, an increase in $C_A$ leads to lower profits and an increase in the optimal $P_A$, while an increase in $P_B$ leads to lower profits but does not change the optimal $P_A$. $C_B$’s lack of impact on $P_A^*$ is due to the assumption that $\theta = 0$ and is not generally true. Together, these two figures demonstrate the smoothness of the profit function over the relevant range for the various parameters. Panel C of Figure 1 graphs the maximum profit, $P_A^*$, and $P_B^*$ as functions of $\theta$ when $C_A = C_B = 0$. The logic of why $P_A^*$ falls with an increase in $\theta$ is obvious from this graph; by charging a lower price for good A the firm can greatly increase the price of good B. Finally, we note that when $C_A = C_B = \theta = 0$, $P_A^* = 50$, $P_B^* = 50$, which is precisely the optimal price in the two independent markets taken separately.

Figure 1. Numerical Results for Sequential Pricing without Discrimination
Panel A: Optimal Profit as a function of $P_A$ for $\theta$ varying from -0.5 (black line) to 1 (light gray line)

Panel B: Optimal Profit as a function of $P_A$ for $C_A$ varying from 0 (black) to 20 (dashed) and $C_B$ varying from 0 (solid) to 20 (dashed).
The above analysis considers the effect of changes in $\theta$; however, as discussed in the introduction, a buyer’s value for two goods may be related in other ways. Specifically, a buyer’s values may be positively or negatively correlated irrespective of the degree of complementarity between the goods. To explore how correlation, $\rho$, may impact profits with sequential pricing, we calculate optimal behavior in a continuum of distributions with varying correlations. While there are many distributions
that one could use for such analysis, any choice is arbitrary unless one has information about a specific set of naturally occurring product markets. Therefore, we choose to focus on distributions that are generated by removing opposing corners from the square domain of \([0,100] \times [0,100]\).\(^7\) This distribution has the advantage of being directly comparable to the results described above for changes in \(\theta\). This type of distribution is also used in related behavioral research by Aloysius, Deck, and Farmer (2009) given its simplicity.

To determine the optimal prices in this environment, we rely upon numerical methods. Specifically, for each level of \(\rho\) we directly calculate the expected profit associated with a given set of prices and then iterate over possible prices.\(^8\) Panel D of Figure 1 plots optimal prices and profits as a function of \(\rho\). Without the ability to price discriminate and no change in the distribution of marginal values of good B due to purchasing good A since \(\theta = 0\), sellers set the same prices for goods A and B. It is also clear from the figure that the optimal prices and profits only depends on \(|\rho|\). When only a few potential customers are removed from the square (i.e. \(|\rho|\) is small) a monopolist lowers its price due to the elimination of some high value customers. This makes the cost of lowering the price smaller since the lower price is being charged to fewer remaining customers. However, as more and more high and low valued customers are removed (as \(|\rho|\) increases) the ability to lower price and serve new customers is reduced and the result is that monopolists begin raising their prices beyond a threshold value of \(|\rho|\) \(\approx 0.58\).

**Case 2: Pricing with Price Discrimination**

\(^7\) A distribution with negative correlation can be achieved by starting with \([0,100] \times [0,100]\) and removing all \((V_A,V_B)\) pairs such that \(|V_A + V_B - 100| > r\) for \(r \in [0,100]\). When \(r = 0\) we have \(\rho = -1\) and the domain is \(V_A + V_B = 100\) and when \(r = 100\) we have \(\rho = 0\) and the domain is the original square. The relationship between \(\rho\) and \(r\) is monotonic but not linear. Similarly, a distribution with a positive correlation can be achieved by starting with \([0,100] \times [0,100]\) and removing all \((V_A,V_B)\) pairs such that \(|V_A - V_B| > 100 - r\) for \(r \in [0,100]\). When \(r = 0\) we have \(\rho = 0\) and the domain is the original square and when \(r = 100\) we have \(\rho = 1\) and the domain is \(V_A = V_B\).

\(^8\) For a given \(r\) (as in footnote 7), we allow each price to range from 0 to 100 in steps of 0.5. For each set of prices we determine what decision a buyer with values \(V_A\) and \(V_B\) ranging separately from 0 to 100 subject to being in the domain of possible values for the given \(r\) would do when facing the given prices. By comparing the expected profits, we determine the optimal set of prices for each \(r\) and we allow \(r\) to vary from 0 to 100 in steps of 2.5 for both the positive and negative distributions. We employ a similar procedure for determining optimal prices and profits under sequential pricing with discrimination as correlation changes as well as the optimal prices and profits with mixed bundling and pure components. Those results are presented later in the paper. For the case of mixed bundling and sequential pricing with discrimination the pricing strategy involves three prices thus raising computational time by a factor of 100/step as compared to the two price strategies. At the level of fineness we use, some computations are in days and thus finer increments quickly extend beyond reasonable computational limits.
In this case the seller will know when setting the price of B whether the consumer has purchased A or not. Formally, the decision is to choose either $P_B|\{q_A=0\}$ or $P_B|\{q_A=1\}$ where $q_A = 0$ if A was not purchased and $q_A = 1$ otherwise. In other words, the seller selects a state contingent price for good B.

**Stage 2: Price of B if $q_A=0$, i.e. $V_A<P_A$**

Since the buyer’s values for A and B are independent and since A is not purchased, the seller’s problem is to maximize

$$E\prod(P_B(q_A = 0)) = (P_B - C_B) \int_{P_B}^{\infty} f_B(V) dV_A$$

Taking the first order condition of (4) and solving yields $P_B^*|\{q_A=0\}$.

For the uniform distribution example (4) simplifies to (4’).

$$(P_B - C_B) \int_{P_B}^{100} \frac{1}{100} dV_B = \left(\frac{P_B-C_B}{100}\right)(100 - P_B)$$

Maximizing (4’) with respect to $P_B$ and solving yields $P_B^*|\{q_A=0\} = \frac{100+C_B}{2}$, the familiar monopoly solution.

**Stage 2: Price of B if $q_A=1$, i.e. $V_A \geq P_A$**

In this case the seller considers the joint valuation of both products when pricing B. In other words, $V_{AB} = (1+\theta)(V_B + V_A)$. Thus, the marginal value of B is $\frac{100P_A}{1+\theta}$ and the consumer will buy B iff $\frac{P_B-V_A}{1+\theta} \leq V_B$. Given this information, the seller chooses $P_B$ to maximize equation (5).

$$\max_{P_B} E\prod(P_B(q_A = 1)) = (P_B - C_B) \int_{P_A}^{100} \int_{P_B}^{100} \frac{1}{100} \frac{1}{100 - P_A} dV_B dV_A$$

Taking the first order condition of (5) and solving yields $P_B^*|\{q_A=1\}$.

Under the assumption of the uniform distribution this can be rewritten as (5’).
Taking the first order condition of (5) and solving for the price of B yields

\[ P_B^* = \frac{100^2(1+3\theta/2) - 100P_A(1+\theta) - \theta P_A^2/2 + (100-P_A)C_B}{200 - 2P_A} \]

Note that once again when \( \theta = 0 \) we get the standard monopoly solution of \( P_B^* = \frac{100+C_B}{2} \).

**Stage 1: The Price of A**

We now need to solve for \( P_A \) given what will occur in stage 2. Specifically we need to know \( E[\Pi] \) when \( q_A = 0 \) and \( q_A = 1 \). Plugging the solution for \( P_B^* (q_A=0) \) and \( P_B^* (q_A=1) \) into (4) and (5) respectively gives the expected profit in each state. The seller will maximize total expected profit over both stages, knowing both the probability that good A will be purchased at a given price and the resulting expected profits in stage 2 based on the follow-up price of good B.

In general this problem is not tractable, but again we can find the solution for the uniform case. Recall that \( P_B^* (q_A = 0) = \frac{100+C_B}{2} \). Computing the resulting profit at the second stage yields \( E[\Pi_B (q_A = 0)] = (P_B - C_B) \int_{100}^{200} \frac{1}{100} \ dV_B \) which can be simplified to \( 25 - \frac{C_B}{2} + \frac{C_B^2}{4(100)} \).

When \( q_A = 1 \), \( P_B^* (q_A = 1) = \frac{100^2(1+3\theta/2) - 100P_A(1+\theta) - \theta P_A^2/2 + (100-P_A)C_B}{200 - 2P_A} \) which yields a corresponding second stage expected profit of \( E[\Pi_B (q_A = 1)] = \frac{[200-2C_B+(300+P_A)\theta]^2}{1600(1+\theta)} \).

Therefore, for the uniform case we have that the first stage profit as a function of \( P_A \) can be written as

\[ E[\Pi] = (P_A - C_A) \int_{0}^{1} \frac{1}{100} \ dV_A + \left( 25 - \frac{C_B}{2} - \frac{C_B^2}{400} \right) \int_{0}^{1} \frac{1}{100} \ dV_A + \frac{[200-2C_B+(300+P_A)\theta]^2}{1600(1+\theta)} \int_{0}^{1} \frac{1}{100} \ dV_A \]

The first order condition is \( -3P_A^2 \theta^2 + 8P_A(-400 + \theta(-500 + C_B) - 125\theta^2) + 4[400C_A(1 + \theta) - C_A^2 (2+\theta) - 2500(-16 - 12\theta + 3\theta^2)] [16000(1 + \theta)]^1 = 0 \). Again the real solutions can be found using Mathematica.

Panel A in Figure 2 shows profit as a function of \( P_A \) for values of \( \theta \) varying from -0.5 (black) to 1 (light gray) assuming that \( P_B (q_A = 0) \) and \( P_B (q_A = 1) \) are set optimally and that \( C_A = C_B = 0 \). Again, profits increase in \( \theta \) while the optimal price of good A decreases with \( \theta \). A panel similar to Panel B in Figure 1 is not presented here because when \( \theta = 0 \), and the value for the two goods are independently distributed, the ability to conditionally price the second good has no effect and thus the two graphs are identical. Panel B in Figure 2 shows the maximum profit, \( P_A^* \), \( P_B^* (q_A = 0) \) and \( P_B^* (q_A = 1) \) as functions of \( \theta \) when \( C_A = C_B = 0 \). From the figure it is clear that \( P_A^* \) falls with an increase in \( \theta \) while \( P_B^* (q_A = 1) \) increases.
in θ and $P_B^*|(q_a=1) = 50$ irrespective of θ. The intuition for $P_A^*$ falling is straightforward, by lowering the price of good A more people will purchase it and the distribution of marginal values for good B will increase. The increase is greater the larger is θ and these higher values results in a higher optimal price of good B for those who purchased good A. The distribution of marginal values for good B for those consumers who do not purchase good A remains uniform on [0,100] and hence the optimal price of good B for these customers remains 50. We also note that when $C_A = C_B = θ = 0$, $P_A^* = P_B^*|(q_a=0) = P_B^*|(q_a=1) = 50$, as in the standard monopoly setting.

Figure 2. Numerical Results for Sequential Pricing with Discrimination

Panel A: Optimal Profit as a function of $P_A$ for θ varying from -0.5 (black line) to 1 (light gray line)
To consider how correlation in the underlying buyer values affects optimal prices and profits (holding $\theta=0$), we again rely upon direct numerical evaluation. The results are presented in Panel C of Figure 2, which has several interesting patterns. First, we note that that expected profit is increasing in $|\rho|$. Second, optimal prices are not monotonic, but are wavy for the same reasons discussed in the no discrimination case. However, good B prices tend to move in opposite directions. When the goods are negatively correlated, buyers who purchased good A are likely to have low values for good B and are quoted low prices while those who did not purchase good A are likely to have high values for good B are
quoted high prices. When the goods are positively correlated, the reverse is true. Further the good B
difference is generally increasing in $|\rho|$. Finally, the price of good A is generally increasing in $\rho$,
although not monotonically for the reasons described above. This upward trend is due to the strategic
value of using $P_A$ to indentify the buyers with high values of good B.

3.2 Impact of the ability to sequentially price

Profits should be at least as great with conditional sequential pricing as without it as the
monopolist could choose to not engage in price discrimination. Clearly a firm would prefer to have the
ability to conditionally price, but how valuable is it and how much are consumes likely to be hurt by the
practice? Further, sequential pricing is an alternative pricing strategy to bundling or pure components
pricing. How does profitability compare across all four strategies? Antitrust authorities worry about the
practice of bundling which dominates pure components in terms of profit (as a mixed bundling
monopolist could always opt to set the bundle price equal to the sum of the individual prices). Should
anti-trust authorities worry about sequential pricing with discrimination too?

Panel A of Figure 3 displays the optimal profits from sequential pricing with and without
discrimination and from mixed bundling relative to the strategy of pure components as $\theta$ varies from -0.5 to 1. Panel B of Figure 3 displays the same comparisons for $\rho$ varying from -1 to 1. From Panel A,
one can observe that bundling is weakly more profitable than pure components (the dashed curve lies
weakly above the zero line) and that sequential pricing with discrimination is weakly more profitable
than sequential pricing without discrimination (the solid dark curve is weakly above the gray curve).
Interestingly, the profit gain from bundling as opposed to components pricing is greater for more
moderate values of $\theta$ while the gain from discriminating in sequential pricing is greater for more
extreme values of $\theta$. Also evident from this figure is that sequential pricing without discrimination is
never more profitable than mixed bundling, but it is more profitable than pure components pricing for
goods that are “weak” substitutes (-0.33 < $\theta$ < 0). Sequential pricing with discrimination is more
profitable than pure components for any $\theta < 0$ and more profitable than mixed bundling when the goods
are “close” substitutes ($\theta < -0.33$).

The intuition for sequential pricing with discrimination being more profitable than mixed
bundling when the goods are close substitutes is that there is little marginal value for the second good

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The author's note: 

9 Profits and $|P_A \ast (q_r=0) - P_A \ast (q_r=1)|$ are nearly symmetric in $|r|$. This asymmetry may be due to the discrete
nature of the numerical estimation.
as compared to the case of complements. Sequential pricing essentially avoids intraseller competition for the two goods whereas bundling attempts to capture the marginal value. When the goods are complements (or even weak substitutes) the reduction in internal competition is not enough to offset the gains from capturing the marginal value of a second purchase, but when the goods are highly substitutable bundling essentially gives the second product away as there is little benefit for purchasing both items whereas sequential pricing avoids the intraseller competition.

From Panel B of Figure 3, we observe that the gain from mixed bundling relative to pure components pricing is decreasing with correlation while the gain from discrimination in sequential pricing is increasing in $|\rho|$. It is also evident from the figure that sequential pricing without discrimination does not perform differently from pure components pricing. This result is intuitive since the purchase of good A does not directly affect the marginal value of good B in this environment and the seller cannot use the purchase history to identify the customer type. What is also clear from the figure is that when the buyer’s value for the two goods are highly positively correlated ($\rho > 0.55$), sequential pricing with discrimination is more profitable than mixed bundling. Mixed bundling breaks down in this environment because it exploits buyers with unequal values, the types that do not occur when values are positively correlated. The tautological results that mixed bundling out performs pure components and sequential pricing with discrimination outperforms sequential pricing without discrimination can be seen in Panel B of Figure 3 from the facts that the dashed curve is weakly above the zero line and the solid dark curve is weakly above the gray one, respectively.

**Figure 3.** Numerical Comparison of Profits from Pricing Strategies Relative
3.3 Rational Buyers

The above analysis compares the results of various pricing strategies applied to myopic buyers. How does this assumption affect the results? First, we note that the practices of pure components and mixed bundling are not affected by this assumption. In both cases, buyers are presented with all information simultaneously and therefore neither party is concerned with extracting information to be used later. Clearly, sequential pricing does involve the revelation of information and therefore the prices that will be observed will be affected by this degree to which the buyer is forward looking. With such buyers the price strategy (in a game theoretic sense) must satisfy both individually rational and incentive compatibility constraints for each buyer type. This constitutes a signaling game in which buyers types may be revealed to some degree and buyers take this into account as they choose their purchase decision.

Reconsidering the simple example presented in the introduction where there are two types of buyers with perfectly correlated values reveals the issue associated with this type of model. In this case the maximum profit from a pure strategy equilibrium would be $2v+V$. A separating equilibrium cannot be supported because if the seller charged $V$ for good A and only a type H would buy it, then the seller would always want to charge $V$ for good B if there was a sale of good A and charge $v$ for good B.
otherwise. This discourages a type H from purchasing good A in order to appear as a type L, and, as a result, the seller will sell no units of good A and only sell good B to type H at a price of V. This encourages the seller to offer v for good A making a sell to everyone and then charging V for good B, which only type H buyers will accept. It is worth noting that in this case a seller would prefer to employ bundling to sequential pricing as it reduces the buyer’s ability to act strategically.

There are mixed strategy equilibria as well, a formal analysis of which is presented in Appendix A. The appendix illustrates the flavor of a model with fully rational buyers. However, extension of this framework to a general distribution of buyers where values could be sub- or super additive is well beyond the scope of this paper. As mentioned in the introduction, it seems unlikely that human buyers are capable of fully recognizing the impact that each purchase they make may have on the array of goods and prices they subsequently may face.

Conclusions

New technologies will enable sellers to engage in new pricing strategies and it is important to anticipate how these strategies are likely to affect sellers and customers. Currently, there is a growing trend in retail markets to track individual items. RFID tags or similar ubiquitous technologies can be used to identify which items a buyer intends to purchase at a given price, just as placing an item in an electronic shopping cart does for an e-tailer. Currently, sellers openly use this information to manage inventory and make recommendations regarding other products. However, this information could also be used to adjust prices on items a shopper is likely to purchase.

The purpose of this paper is not to compare specific technologies for pricing strategy implementation, nor is it about the feasibility of the practice with current technologies. This research is meant to be forward looking in identifying how such technological changes may affect pricing strategies when such capabilities are feasible. “Marketers recognize that marketing programs intended for different segments—be they products, prices, or advertising messages—must embody different benefits, and if these benefits are well chosen then consumers will themselves choose what they are supposed to choose” (Moorthy 1984). What are the likely implications of sellers being able to set prices sequentially and discriminate based upon the previous actions of a buyer? As a first step, this paper presents a theoretical model that can be used to answer this question for monopoly markets. The results indicate that the ability to set prices sequentially, absent the ability to discriminate, increases profits relative to a pure components framework where the monopolist sets a price for each good simultaneously when the
goods are weak substitutes. This is due to sequential pricing reducing the intraseller competition between products sold by the same retailer. Further, sequential pricing with conditional pricing, a form of behavior based price discrimination, is more profitable than mixed bundling when the goods are either close substitutes or when the goods are highly positively correlated.

This research is only a first step in studying sequential conditional pricing. One important area for future exploration is how buyers will react to such practices. For example, Coca-Cola developed a vending machine which would adjust price with temperature. However, the negative attention this attracted thwarted implementation. More generally, Kahneman, et al. (1986) report that people deem it “unfair to exploit shifts in demand by raising prices.” (p. 728). As pointed out by Acquisti (2006, p2.),

Nobody likes to pay for the same product more than the amount the other person spent. Faced by intrusive information policies and price discrimination strategies, however, consumers can decide to bypass the seller’s tracking attempts through privacy enhancing and anoyymizing technologies, or to avoid the seller altogether.

Of course, people knowingly pay different prices on everything from movie tickets (with discounts for the elderly) to college tuition (with scholarships for students with high test scores). In part, popular acceptance is affected by how the practice is presented and how long the practice has been used. One can imagine the reaction if retailers charged a “penalty” for buyers who failed to cut a notice out of a flyer, but people are willing to accept that a “discount” is only applied to those who have clipped coupons even when the flyer was only sent to a targeted audience.

Would consumers stand for being monitored or will their reaction possibly trigger intervention by the Federal Trade Commission (Weiss and Mehrotra 2001)? The very possibility of regulation may cause firms to alter their strategies. However, the use of cookies is widespread and the practice allows sellers to monitor customers and price discriminate. Many retailers, including Albertson’s through its Preferred Shopper Program, offer discounts for people voluntarily identifying themselves to be tracked across visits. Casual observation indicates that people, including one of the authors of this paper, are willing to put up with the privacy invasion for a savings of a few dollars. How would buyers attempt to strategically shop? Would people make several trips into a store from the parking lot or have friends buy products to save a few cents? The answer is probably not so long as the transaction and contracting

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costs are greater than the savings, but those few cents could generate a large increase in profit for a retailer serving many customers. Would buyers be less likely to visit sellers who engage in sequential conditional pricing? Perhaps they would if the practice is not universally adopted. Ironically, Albertson’s used to advertise that they did not engage in a Preferred Shopper Program.

It is important to note that the ability to set sequential prices does not only help the seller, but it is also potentially beneficial to buyers. In the numerical example presented in the introduction, under sequential pricing with discrimination, person $L$ is able to purchase good $B$, which would not be the case otherwise. It is also easy to imagine situations where the $B$ product is one with which the buyer was unfamiliar initially. The desire to generate profits will lead sellers to make buyers aware of more potentially valuable products. Ayres (2007) lists many naturally occurring examples of collaborative filtering generating recommended products that the buyer would not have been aware of otherwise, thus increasing a buyer’s overall satisfaction. The tactical measures that retailers may employ as a result of this research should be viewed in the context of contemporary initiatives by firms such as Sam’s Club to fully automate the retail shopping experience (Weier 2008). Kourouthanassis and Roussos (2006) describe such a ubiquitous technology-enabled retail environment in which a shopper may place products in a smart-cart and check out automatically. They describe a study in which shoppers in such an environment perceived customized offers as improving the effectiveness of the shopping experience. The real-time information about customer’s preferences provided the ability to identify their emerging needs.
References


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Appendix A. Rational Buyers in a Simple Environment

A buyer is considering purchasing two goods A and B from a monopolist who offers the two goods sequentially and sets the price of good B after observing the buyer’s purchase decision for good A. Suppose a buyer is type $H$ (value $= V$) with probability $p$ and is of type $L$ (value $= v$) with probability $1 - p$. Assume $pV > v$. A seller chooses prices for each good denoted $P_A$ and $P_B$ and suppose that $P_A = V - \Delta > v$ since no buyer would purchase good A at $P_A = V$ if there was any positive probability that $P_B < V$.

**Seller Strategy:** If buyer buys at $P_A$, the seller knows the buyer is type $H$ and sets $P_B = V$ so $\pi = V - \Delta + V, CS = V - \Delta$ for type $H$.

If the buyer does not buy good A, the seller will need to update her belief of the probability the buyer is type $H$ depending on the rational buyer’s strategy. Define this updated probability to be $q$. $P_B = V$ iff $q V > v$. Where $q$ is determined in equilibrium.

**Buyer Strategy:** If buyers separate: i.e, type $H$ buys at $V - \Delta$ with probability 1, then

$q = 1$ if good A is purchased and $P_B = V$, $CS = V - \Delta$

$q = 0$ if good A is not bought and $P_B = v$, $CS = V - v$ which is greater

$\Rightarrow$ type H buyer will not buy good A and a separating equilibrium cannot be supported.

**Mixed Strategy Equilibrium:**

If seller observes no purchase, then the seller chooses $P_B = V$ with probability $\alpha$ and $P_B = v$ with probability $1 - \alpha$. A Type $H$ buyer is indifferent between buying at $V - \Delta$ and not iff $\Delta = (1 - \alpha)(V - v)$ . This identifies the equilibrium probability for the seller, $\alpha^*$.

The Seller will be indifferent (and $\Rightarrow$ choose $P_B = V$ with probability $\alpha$) iff $\beta \equiv$ probability type $H$ buys good A $V - \Delta$ which implies $q = \frac{P(1 - \beta)}{P(1 - \beta) + 1 - p}$ is the updated probability buyer is of type $H$ given they did not buy good A. Therefore, if the seller charges $P_B = V$ when they observe that good A is not purchased then $\pi = \frac{P(1 - \beta)}{P(1 - \beta) + 1 - p} \cdot V$ and if they charge $v$ they get a sale with probability 1. The seller is indifferent iff $\frac{P(1 - \beta)}{P(1 - \beta) + 1 - p} \cdot V = v$ which implicit defines $\beta^*$. $\alpha^*, \beta^*$ are equilibrium mixed strategy probabilities and the seller’s profit would be $\pi = p[\beta(V - \Delta + v) + (1 - \beta)[(1 - \alpha)v + \alpha V]] + (1 - p)((1 - \alpha)v)$.