Econ 5233 Cary Deck Name___________
Exam 3 - 2008 Score___________

Q1. Consider a matrix A= $\begin{bmatrix} 2 & -1 & 2 \\ 1 & a & 3 \\ 6 & 2 & 0 \end{bmatrix}$ and the vector **x**= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

a. Find $\partial \mathbf{x}'A\mathbf{x}/\partial \mathbf{x}$.
b. Find $\partial \mathbf{x}'A\mathbf{x}/\partial a$.
c. Find $\partial |A|/\partial a$.

Q2. Find the solutions to the following optimization problems making sure to verify the second order conditions.

a. $\max_x f(x) = x^3 + 3x^2$ on the interval [-3,3]

b. $\min_{x,y,z} f(x,y,z) = x^2 + y^2 + z^2$ subject to x - y =1 and $y^2 - z^2 = 1$

c. $\max_{x,y} f(x,y) = x^{1/2} + y^{1/2}$ subject to $x+y \leq 20$ and $x \geq 5$

Q3. Evaluate the following expressions

a. $f(x) = \frac{(x^2-y)^2}{(y-3)}$. Find f_{yx}.
b. $\int x2^x\, dx$
c. $f(x) = (x^r+y^r)^{1/r}+17$ Find f_x.
d. $2ye^x = e^{xy} + 2ex^3 + x^{-2}y^4 +$. Find dy/dx.
e. $\int \frac{3x+2}{2x^3+4x} dx$
f. $\frac{\partial}{\partial x}\int_1^5 x^2 y\, dy$
g. $\iiint_R x + yz\, dzdydx$ where R is the region defined by $x \in [0,2]$, $y \in [x,2]$, $z \in [x,y]$

Q4. Consider the following first order linear difference equations $X_t = .9X_{t-1} + .2$ and $Y_t = .1Y_{t-1} + .5$.

a. Solve the equation so that you can write X_t as a function of X_0, t, and the steady state.
b. Draw a phase diagram for Y_t and show what would happen if $Y_0=1$. Make sure to identify the steady state on your picture.
c. Suppose that we had the following system instead: $X_t = .9Y_{t-1} + .2$ and $Y_t = .1X_{t-1} + .5$. Find the steady states of this system and determine if they are stable by diagonalizing the system.

Q5. Find the 2^{nd} order Taylor Approximation to $f(x,y) = x^5y + xy^3 + 7x + 4y$ at (-1,1)

Q6. Consider the following matrices

A= $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$, B= $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$, C= $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \end{bmatrix}$, D= $\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 2 & 3 \end{bmatrix}$, E= $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

a. What is the trace of CD?
b. What are the eigenvlaues of B?
c. What is the determinant of A?
d. What are the dimensions of (DBE)'

Q7. Determine if the following functions are concave, convex, quasiconave, or quasiconvex

a. $f(x)=x^3-4x$ for $x \geq 2$
b. $f(x,y)=x^2+y^3+2xy$ at (1,1)

Q8. Solve the following systems of equations.

a. using Cramer's Rule
   x+2z=25
   2y-z=3
   x+y-z=-1

b. by calculating $A^{-1}b$.
   x-2y=-3
   2x+2y+z=2
   -x+y+z=4

Q9. Bob has utility given by $u(x,y)=x^a y^b$. Bob has income I and faces prices of Px =3 and Py = 5. Bob cannot purchase negative amounts of either good.

a. Find the optimal amount of good x and y that Bob should purchase and give an economic interpretation of the value of Lagrange multiplier for the budget constraint.
b. Find the comparative static effects for a change in b for Bob's optimal bundles of x and y.
c. Find Bob's indirect utility functions V(P,I), which is a value function.
d. Use the envelope theorem to determine how Bob's utility would change if his income changed.

Q10. Consider the function f(x,y)=6(1-y) for $0 \leq x \leq y \leq 1$ and 0 otherwise.

a. Show that f() is a proper probability distribution function.
b. If $z = x^2+y+1$, what is the expected value of z?
c. What is the probability that x < 0.2?