

1. Consider a matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and the vector  $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ .

a. Find  $\partial \mathbf{x}' A \mathbf{x} / \partial \mathbf{x}$ .

b. Find  $\partial \mathbf{x}' A \mathbf{x} / \partial A$ .

2. Find the following derivatives.

a.  $f(x) = \frac{2x^2 - x}{x^2} + x$ . Find  $\frac{d}{dx} \left( \frac{df}{dx} \right)$ .

c.  $f(x) = \ln(x^3 + 2x) + 17$ . Find  $f_x$ .

b.  $f(x) = \frac{(x^2 - x)^2 (x^3 - 3)}{(2x + 7)}$ . Find  $f'(x)$ .

d.  $f(x, y) = x^3 + x^2 y^4 + 2y + e^{xy}$ . Find  $f_{xyy}$ .

e.  $\ln(xy) = x^3 + y^3 + 2x^2 y^2$ . Find  $dy/dx$ .

3. Evaluate the following integrals.

a.  $\int_1^3 x^2 + 2x \, dx$

b.  $\int x(2x^2 + 3)^{10} \, dx$  (hint use substitution)

c.  $\int x^2 e^{3x} \, dx$  (hint: use integration by parts twice)

d.  $\int_1^3 \int_{1-x}^{x^{1/2}} x^2 y \, dy \, dx$

e.  $\int_1^4 \int_2^x x^3 + \ln(y^3 + 2^x) \, dx \, dy$

4.  $p(x) = 1/x^2$  for  $x \in [2/3, 2]$  and 0 elsewhere.

a. Show that  $p(x)$  is a proper probability distribution function.

b. What is the probability that  $x < 1$ ?

c. What is the variance of  $x$ ?

d.  $z = a + b^2 x$ . What is the expected value of  $z$ ?

5.  $f(x, y, z) = x^2 y^2 + y^2 z^2$ .

a. Determine if this function is concave, convex or neither at  $(1, -1, 1)$ .

b. Determine if this function is quasi concave, quasi convex, or neither at  $(1, -1, 1)$ .

6. What are the dimensions of the following matrix expressions if  $A$  is  $3 \times 3$ ,  $B$  is  $3 \times 1$ , and  $C$  is  $2 \times 3$ ?

a.  $A^{-1} B B'$

b.  $B(CB)'C$

c.  $BAC'$

d.  $(CAB)'$

7. Solve the following system of equations using Cramer's rule.

$$2x + y - z = 6$$

$$2x - y = 1$$

$$3y - 4z = 5$$

8. Bob has utility given by  $u(x, y) = x^3 y^2$ . Currently Bob has the bundle  $(4, 8)$ .

a. Determine if this function is homogeneous and if so, to what degree.

b. Find the slope of the Bob's indifference curve at  $(4, 8)$ . The indifference curve is the set of all points that generate the same utility (i.e. a level set).