

Q1.  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & -2 \\ 6 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ ,  $D = \begin{bmatrix} -1 & 1 \\ 0 & 2 \\ -2 & 0 \end{bmatrix}$ ,  $E = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Find the following values if possible. If it is not possible explain why.

- |            |                   |             |
|------------|-------------------|-------------|
| a. $A^T C$ | d. $2B + E^T A E$ | g. $A^{-1}$ |
| b. $ CD $  | e. $BD$           | h. $B^{-1}$ |
| c. $DC$    | f. $E^T C E$      |             |

Q2.  $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$

- Show that the trace of A is the sum of its eigenvalues.
- Show that the determinant of B is the product of its eigenvalues.
- Find a matrix P such that  $AP = PD$  where D is a diagonal matrix.
- What are the eigenvalues of  $B^{-1}$ ?

Q3. Consider the following system of equations

$$3x + 2y = 6$$

$$x + y + z = 10$$

$$2y - z = 1$$

- Write out this system of equations in  $A\mathbf{x} = \mathbf{b}$  form.
- Use Cramer's rule to determine the values of x, y, and z.
- Create an augmented matrix  $A|\mathbf{b}$  and use elementary row operations to find the solution for  $\mathbf{x}$ .
- Find  $A^{-1}$  and use it to determine the solution  $\mathbf{x}$ .

Q4. Determine the rank of  $A = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 7 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ .

Q5. For each of the following matrices determine if the matrix is positive definite, positive semidefinite, negative definite, negative semidefinite, or indeterminate using principal minors.

a.  $A = \begin{bmatrix} 8 & 4 & 0 \\ 4 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$       b.  $B = \begin{bmatrix} -2 & 2 & 1 \\ 2 & -5 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

Q6. In a competitive market the demand for a good is given by  $P = A - BQ$  and the supply curve is given by  $P = Q + T$  where T is a per unit tax.

- Using matrix algebra, determine the reduced form equations for P and Q.
- Find the comparative static effect for an increase in T.
- Find the comparative static effect due to a decrease in B.

Q7. B is a 2x2 matrix. Show that  $B^T B$  is positive definite if B is invertible.