

You must answer 60 points worth of questions. Place an X on the blank next to the questions you want to count towards your grade.

___Q1. Linear Systems (15 points) Consider the following linear system of equations

$$\begin{aligned}2x+3y+z&=5 \\ x+y-z&=-1 \\ -x-y+2z&=3\end{aligned}$$

Put this system into the form $A\underline{x}=\underline{b}$ and solve for x , y , and z by using Cramer's rule, calculating $A^{-1}\underline{b}$, and Gaussian elimination.

___Q2. Derivatives (10 points) Calculate the following derivatives

- f' when $f(x)=(x^2+2x^3)^3/(\ln x)$
- dy/dx when $x^2y^2=\ln(xy)+2y$
- $\partial^3f/\partial x\partial z\partial z$ when $f(x,y,z)=x^3y^2z^3+y^xz$
- the total derivative dz when $z=(x^4y+y^4x)^2$
- f_{xx} when $f(x,y)=xe^y y^3 \ln(y)/(y+3)+x^2y$

___Q3. Matrix Operations (15 points)

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -2 & -5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Determine the following values or explain why it cannot be calculated.

- a. $(AB)^{-1}$ b. $AD+C$ c. $\text{rank}(BA)$ d. $BA+DD'$ e. $(B'D)'C$

___Q4. Integrals (10 points) Evaluate the following integrals

- $\int \frac{\ln(x^3)}{x} dx$
- $\int_2^y (2xy + e^y + x^2) dx$
- $\int_1^\infty x^{-4} dx$
- $\int \ln x dx$ (hint: use integration by parts)
- $\iiint_R xy + z dV$ where R is the region where $x \in [0,1]$, $y \in [x,z]$, $z \in [0,2]$

___Q5. Concavity (10 points) For both of the following functions determine if it is concave, convex, quasiconcave, or quasiconvex over the given domain.

a.
$$f(x) = \begin{cases} 0 & \text{if } x < 2 \\ (x - 2)^2 & \text{if } 2 \leq x \leq 5 \\ 6x - 21 & \text{if } x > 5 \end{cases}$$

b. $f(x,y,z) = x^2y^2z^2 + x + y + z^2$ at $(2,0,1)$.

___Q6. Eigenvalues (5 points) $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Find the eigenvalues of A and construct a matrix P such that $A = PDP^{-1}$ where D is the diagonal matrix with the eigenvalues on the diagonal.

___Q7. Statistics (5 points)
$$p(y) = \begin{cases} .2 & \text{if } y \in [0,2] \\ .3 & \text{if } y \in (2,4] \\ 0 & \text{else} \end{cases}$$

Show that $p(y)$ is a probability distribution. Calculate $V(y)$ and $E(z)$ where $z = 3y^3 + 5$.

___Q8 Matrix Calculus (5 points) $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 1 \\ 5 & 1 & 2 \end{bmatrix}$ $\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Find $\partial A\underline{x} / \partial \underline{x}$ and $\partial \underline{x}'A\underline{x} / \partial \underline{x}$.