

Exam 4

1. Find the steady state values of the following system of difference equations that describe how capital (K) and labor (L) evolve in an economy and determine the stability of the system.

$$\begin{aligned} K_t &= K_{t-1} - L_{t-1} + 1 \\ L_t &= 2K_{t-1} + 4L_{t-1} - 2 \end{aligned}$$

Suppose instead that the economy was described by the following system. Determine the steady state and the stability of the dynamic process for capital and labor. Solve the difference equation for capital assuming the initial capital stock was $K_0=5$

$$\begin{aligned} K_t &= .3K_{t-1} + .1K_{t-2} + 3 \\ L_t &= .4L_{t-1} + .2L_{t-2} + .1L_{t-3} + 3 \end{aligned}$$

2. Define quasiconcave and concave. Draw a curve that is not concave anywhere but is quasiconcave everywhere and give a graphical justification for your example. (That is, show on your graph why the function is not concave but is quasiconcave.) Explain how you can use determinants to verify that a function, $f \in C^2$, is concave or quasiconcave.

3. Consider the following system of equations

$$\begin{aligned} 2x + 4y + z &= 10 \\ x + y &= 2 \\ -2x + y - 2z &= 5 \end{aligned}$$

- Write the system in matrix notation.
- Solve the system using Cramer's rule.
- Solve the system by finding the inverse of the coefficient matrix.

4. Show that the midpoint of a linear demand curve ($P=A-BQ$) is unit elastic ($\epsilon=-1$). Recall that elasticity is defined as the percentage change in quantity for a percentage change in price.

5. Evaluate the following derivatives

- $f(x) = \frac{x}{e^{2x}}$, find $f''(x)$
- $f(x,y) = \ln(xy) + e^{-x+y}$, find f_{xy}
- $\ln y = Ay^2 + Bx^3 + Cxy + D$, find dy/dx

6. Consider the problem faced by a firm that makes outputs for two markets and has a cost structure in which there is an interaction between the quantity produced for each market. Specifically, the firm wants to $\max_{q_1, q_2} \pi(q_1, q_2) = (A - q_1)q_1 - (B - q_2)q_2 - c q_1 - c q_2 - \gamma q_1 q_2$. Find the firm's value function which expresses profit as a function of A, B, c, and γ . What is the comparative static effect on q_1 and q_2 for a change in γ ?

7. Evaluate the following integrals.

a. $\int_1^{\infty} \frac{1}{x^2} dx =$

b. $\int_0^2 \frac{x^2+1}{x^3+3} =$

c. $\int_0^2 \int_{-3}^3 |x|y \, dx dy =$

8. Consider the following matrices $A = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ -1 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

Find the following

- a. AB
- b. BA
- c. AC
- d. $\text{tr}(BA + C)$

9. maximize $f(x,y,z) = xz + yz$ subject to $y^2 + z^2 = 1$ and $x + z = 3$.

10. A person has preferences expressed by $u(x,y,z) = x^a y^b z^{1-a-b}$ with $a, b > 0$. If the person faces prices of P_x , P_y and P_z and has income I , determine the optimal amount of each good the person should purchase assuming they cannot purchase a negative amount of any good, that is, find their Marshallian demand. Also find the indirect utility function which gives the maximum utility a person can achieve for a given set of prices and income.

11. maximize $f(x,y) = -2x^2 - 2y^2 + 2xy + 9y$ subject to $4x + 3y \leq 10$, $y - 4x^2 \geq -2$, $x \geq 0$, $y \geq 0$.