

1. Derivatives

a. $f(x) = a^{7x}$. Find $df(x)/dx$

e. $f(x,y) = x^2y + xy^2 - xy + 4$. Find f_{xy} .

b. $y = x^2y^3 + x^3y^2$. Find dy/dx

f. $f(x) = \ln(x^4 + 2)^2$. Find $f'(x)$

c. $f(x) = e^{x^2+3x-2}$. Find $f'(x)$

g. $f(x,y) = (x+y^2)/(x+y)$. Find f_{yx}

d. $f(x) = \frac{x}{\ln x}$. Find $f''(x)$

h. $f(x,y) = x \ln(y) + y \ln(x)$. Find f_{xy} .

2. Convexity and Quasiconvexity.

a. Define convexity and quasiconvexity.

b. For what values of x is $x^3 - 3x$ convex? quasiconvex?

3. The Stone-Geary utility function is $u(x,y) = a \ln(x-x_0) + b \ln(y-y_0)$. Find the marginal rate of substitution for this utility function (that is, find the slope of the indifference curve).

4. Are the following equations homogenous? If so, to what degree?

a. $f(x,y) = x^4/(\beta y) + y^5/(x^2 + \delta y^2)$

b. $f(x,y) = e^{y/x} + \beta x + \delta y$

5. Suppose that Demand is given by $P = Q^{-\alpha}$. Find the elasticity of demand ($\varepsilon =$ percent change in Q divided by percent change in P).

6. A competitive firm (P is not a function of Q , that is the firm cannot influence the price by adjusting quantity) has a production function given by $Q = L^\alpha K^{1-\alpha}$ with $\alpha < 1$. In the short run capital is fixed at $K = K_1$, so the only choice variable the firm has available is labor L . The cost of using the input bundle (L, K) is $wL + vK$. Write out the firm's profit function. Determine the optimal amount of labor as a function of the parameters, P , w , v , K_1 , and α [remember to check the second order condition]. Find the comparative static effects of changes in P , w , and v .