

### Day 3 Problems

1. Consider the following matrices.

$$\text{a. } A = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$$

$$\text{b. } A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

$$\text{c. } A = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$$

$$\text{d. } A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{e. } A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & -1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of each matrix and show that the product of the eigenvalues equals  $|A|$  and that the sum of the eigenvalues =  $\text{tr}(A)$ .

2. For a and b in #1, show that A is similar to a diagonal matrix D where the diagonal entries are the eigenvalues of A by finding a matrix P such that  $AP = PD$ .

3. Determine if the following matrices are positive definite, negative definite, positive semidefinite, negative semidefinite, or indefinite by 1) calculating the eigenvalues and 2) using determinants.

$$\text{a. } \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}$$

$$\text{b. } \begin{bmatrix} 3 & -5 \\ -5 & 4 \end{bmatrix}$$

$$\text{c. } \begin{bmatrix} -5 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\text{d. } \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{e. } \begin{bmatrix} 2 & 4 & 0 \\ 4 & 8 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$