

Day 2 Problems

1. Consider the following systems of equations.

a. $x - 3y + 6z = -1$
 $2x - 5y + 10z = 0$
 $3x - 8y + 17z = 1$

c. $x + y + z = 5$
 $2x + y = 15$
 $x + y + 3z = -5$

b. $x + y + z = 0$
 $12x + 2y - 3z = 5$
 $3x + 4y + z = -4$

Write each system in the form $Ax = b$. Solve each system by 1) calculating $A^{-1}b$, 2) using Cramer's rule, and 3) using Gaussian elimination.

2. Two firms sell differentiated products according to a Cournot model. The profit to firm i is given by $(A_i - b_{ii}q_i - b_{ij}q_j)q_i - (c_i + t_i)q_i$, $j \neq i$. The optimal quantity condition for firm i is given by $A_i - 2b_{ii}q_i - b_{ij}q_j - c_i - t_i = 0$ (This is a first order condition from profit maximization). C denotes the per unit cost for each firm and t is a per unit tax. Since each firm should be making an optimal decision, set up the problem in $Ax=b$ form. Solve for the optimal quantity each firm should produce. How would your answers change if the tax rate on firm 1 increased? How does the direction of the change in quantity depend on the sign of the parameters?